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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

A. W. WUNDHEILER, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ĭ	′	è	yu	ya

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A Critical Review of the World Literature in Applied Mechanics

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Communications

The Translation Project of American Mathematical Society

Under a contract with the Office of Naval Research, 125 photo-offset copies of English translations of selected papers in pure and applied mathematics will be distributed to libraries, and a number of them will be available for loan at the A. M. S. library. A list of planned translations (all of them from the Russian) has been distributed, and individual orders for copies of the yet unissued items will be filled (at about one cent a page). The list can be obtained from A. M. S., 531 West 116th Street, New York 27, N. Y. It includes items covered by our Reviews **1: 1247, 1383; 2: 86.**

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Office de Documentation Bibliographique et de Diffusion du Livre Français (33, Rue de Seine, Paris, France), invites bibliographic queries and also research assignments.

Concerning Rev. 204, February 1949, "Drag coefficients of steel spheres entering water vertically" by A. May and J. C. Woodhull.

The reviewer stated that the authors could not expect their $1/V$ versus t plots, such as that of Fig. 3, to be straight lines since "their final data show that the drag is proportional to $V^{2.13}$." Here the reviewer has missed the point described on page 1111 and exemplified in Fig. 4, that the $1/V$ values were adjusted before plotting so as to remove the dependence of C_D on velocity.

The reviewer stated also that "the drag coefficient should be analyzed as a function of such variables as Reynolds' number, Froude's number, cavitation parameter, ratio of depth to diameter, and ratio of water to sphere density," instead of Reynolds' number and Froude's number only. Our paper clearly states that only steel spheres were used so that the density ratio was always the same, and that the cavitation parameter was kept practically constant by reducing the air pressure. Determination of the dependence on depth was beyond the accuracy then attainable.

Albert May, USA

May's first objection is justified because the adjustments to the data do in fact correct the drag during each shot back to the conditions at entry. This is done by using the final dependence of drag on velocity to correct the individual shots. The reviewer's confusion arose from the repeated statement in the paper that the drag coefficients so obtained "correspond only to the inertial forces" or "only to the v-squared-law drag forces," and that all other forces are absent or their effects have been eliminated. This is untrue because the coefficients so obtained are correlated on the basis of Reynolds' and Froude's numbers, indicating directly a dependence on noninertial or non-v-squared-law forces.

Concerning May's second objection, one of the largest adjustments made to the drag coefficient was for the effect of buoyancy. Following May's picture that the buoyancy acts on the wetted face, the correction to the drag coefficient can be expressed as a term depth-to-diameter parameter divided by Froude's number. Thus a variation with depth to diameter was actually found and approximated by a linear law in spite of May's protestations to the contrary. That the depth-to-diameter parameter plays a role in addition to that visualized by May is indicated by the fact that he had to reduce his correction arbitrarily almost 50% to get a sensible fit to the data. The pertinent question is whether a better fit could have been obtained if a preconceived law had not been forced onto the data. Next, the cavitation parameter was not kept constant as much as it was kept small. There is not a priori reason to believe that this removes its influence any more than keeping the reciprocal of the Reynolds' number small removes the effect of friction, which is of course false. Further, in the data that were used, there is a correlated variation of atmospheric pressure with diameter so that the effects of diameter and cavitation are not separable. Lastly, May is correct in stating that the density ratio was held constant during the experiments. The reviewer should have explicitly stated that while the density ratio should be used to correlate drag data in the general case, it was a fixed parameter in the present case. Francis H. Clauser, USA

The cavitation parameter varied over a range of about 4×10^{-6} to 16×10^{-5}. Dependence of C_D on it was looked for but not found, although it was easily noted when cavities closed at high pressures and the cavitation constant was perhaps 10^3 times as large. Insufficient data were available to permit correlation with cavitation constant, temperature, Weber's number, or Mach number. The authors believe that these were adequately considered in a study of necessarily restricted scope. Albert May

Theoretical and Experimental Methods

(**See also Revs. 11, 14, 17, 19, 28, 29, 91, 97, 114, 129**

1. Hans Ertel, On the graphical construction of trajectories in flow fields (in German with Russian summary), Z. angew. Math. Mech. 28, 285–289 (1948).

A new method of successive approximations for the solution of a system of equations to evaluate the small coordinate difference of two neighboring points of a trajectory, is stated and transformed into a graphical method for the point-by-point construction of trajectories. C. C. Lin, USA

2. W. J. Duncan, Galerkin's method in mechanics and differential equations, 33 pp. (Aug. 1937), **The principles of the Galerkin method,** 24 pp. (Sept. 1938), Rep. Memo. aero. Res. Counc. Lond. nos. 1798 and 1848 (published 1947).

A detailed exposition is given of Galerkin's method of approximate solution of boundary-value problems in differential equations. For problems like cantilever beams, the method can be interpreted as substituting a system of a finite number of degrees of freedom for a continuum. Galerkin's method leads then to Lagrange equations of the system. It is also shown in the second paper that for a conservative system Rayleigh's principle and Galerkin's method necessarily lead to identical results. Hence the errors in the frequencies of a conservative system obtained by Galerkin's method are of the second order relative to those in the displacements. Galerkin's method is applicable also to non-

conservative systems. Various static and dynamic problems on elastic bodies are worked out in detail as illustrative examples in both papers. S. S. Shu, USA

3. Edmond Lahaye, A method of solving differential equations (in French), Bull. Acad. Belg. Cl. Sci. 34, no. 12, 851–862 (1948).

Newton's method of successive approximations is extended to the solution of nonlinear systems of ordinary differential equations. In order to carry out a solution by this procedure it is necessary to begin with an approximate solution of the system of equations in question. The author proposes a scheme for choosing such an approximate initial solution. He proceeds to demonstrate that this choice will, under rather general conditions, yield successive approximations which will converge to the true solution of the system. Each successive iteration involves the solution of a linear system of differential equations.

H. Polachek, USA

4. A. E. Carter and D. H. Sadler, The application of the National Accounting Machine to the solution of first-order differential equations, Quart. J. Math. 1, 433–441 (Dec. 1948).

The authors employ Milne's formula for approximate quadratures. Their method uses the National Accounting Machine (a printer-adder with six adding mechanisms, direct subtraction on two), an ordinary calculating machine, tables, and mental arithmetic. Except for extrapolation and subsequent correction, all operations soon become almost automatic, and the computer can concentrate on the flow of the solution.

A. W. Rankin, USA

5. Luigi Amerio, Relations between the method of multiple Laplace transform and the method of M. Riesz for the integration of hyperbolic type equations (in Italian), R. C. Accad. Lincei (8), 5, 313–319 (Dec. 1948).

This note deals with the solution of Cauchy's equation of order $2k$ in m variables:

$$E_k(u) = (\partial^2/\partial x_1^2 - \partial^2/\partial x_2^2 - \ldots - \partial^2/\partial x_m^2 + \lambda)^k u = f$$

where λ is an arbitrary constant and f a given function of $x_1, x_2, \ldots x_m$ in a domain $S_m(x_1, x_2, \ldots x_m)$, by means of the multiple Laplace transform without making any hypothesis on the behavior at infinity of the solution. The method followed is an extension of a procedure used by the author in a previous paper [R. C. Circ. Mat. Palermo, 63 (1942)]. Ernesto Saleme, USA

6. Josef Meixner, Series expansions of Sieger's type for the spheroid functions (in German), Arch. Math. 1, no. 6, 432–440 (1948/49).

For the wave equation in spheroidal coordinates, separation of variables leads to the ordinary differential equation,

$$(1)\quad (\xi^2 - 1)f'' + 2\xi f' + \left(\gamma^2\xi^2 - \frac{\mu^2}{\xi^2 - 1} - \lambda\right)f = 0.$$

When $\mu = \pm\frac{1}{2}$, Equation (1) can be transformed into Mathieu's equation. For this particular case, series expansions in terms of products of Bessel functions were given by Sieger. The present author derives similar expansions for Equation (1), for general μ and for values of λ which admit solutions which are bounded in the interval $(-1, 1)$. By the use of an integral relation between different solutions of (1) it is shown that

$$(2)\quad f(\xi) = (\xi^2 - 1)^{\mu/2} \sum_i^\infty (\mu + j + \tfrac{1}{2}) A_j J_{\mu+j+1/2}(r) B_{\mu+j+1/2}\omega$$

satisfies (1), where

$$r = \tfrac{1}{2}\gamma[\xi - (\xi^2 - 1)^{1/2}], \quad \omega = \tfrac{1}{2}\gamma[\xi + (\xi^2 - 1)^{1/2}],$$

$J_{\mu+j+1/2}$ is a Bessel function of the first kind, and $B_{\mu+j+1/2}$ is any other Bessel function or again $J_{\mu+j+1/2}$. The coefficients A_j are integrals involving Gegenbauer functions. Recurrence formulas are established for these coefficients and the asymptotic behavior of the series is discussed. [Compare M. J. O. Strutt, *Lamésche, Mathieusche und verwandte Funktionen etc.*, Ergebnisse der Mathematik, I, 3]. A. Robinson, USA

7. L. A. Lyusternik, Remarks on the numerical solution of boundary problems for Laplace's equation and the calculation of characteristic values by the method of networks (in Russian), Trav. Inst. Math. Stekloff 20, 49–64 (1947).

The method of successive approximations as used in solving the vector equation (1) $x = Ax + y$, where y is known, x is an n-dimensional vector and A is an n-dimensional matrix, consists in starting with an arbitrary vector x^0 and forming successively the system of vectors $x^1, \ldots, x^k, \ldots$, where (2) $x^k = Ax^{k-1} + y$. If the sequence x^k converges to the vector x, it is said to be the solution of Equation (1). However, the convergence may be very slow, in which case the problem is to extrapolate the approximations $x^1, \ldots, x^k$ already found so as to make the convergence more rapid without directly substituting in Equation (2).

The author first considers the simple case when A is a symmetric matrix. Letting $\lambda_1, \ldots, \lambda_k$ be the characteristic values of A, and $u_1 \ldots, u_k$ the orthonormal system of the characteristic vectors, it follows that $x^k - x = \Sigma\lambda_i^k c_i u_i$. Thus, in particular, when a single greatest characteristic value λ exists,

$$x^k - x \sim \frac{1}{1 - \lambda}(x^k - x^{k+1}) \sim \frac{1}{1 - \lambda^2}(x^k - x^{k+2}),$$

and therefore x can be found when x^k, x^{k+1}, x^{k+2} are known. The case when two greatest (in absolute value) λ's exist is also considered.

As an illustration the method is applied to the numerical calculation of the solution of plane boundary problems of Laplace's equation by the method of networks. Here the problem reduces to finding a function u which satisfies, in addition to certain boundary conditions, the equation $u = Du$, where

$$Du = D^h u = \tfrac{1}{4}h^2\nabla^h u + u,$$

$$\nabla u = \partial^2 u/\partial x^2 + \partial^2 u/\partial y^2,$$

$$\nabla^h u = 1/h^2[u(x + h, y) + u(x - h, y) + u(x, y + h) + u(x, y - h) - 4u(x, y)].$$

A numerical example is given.

The method is also applied to the Sturm-Liouville equation. *Courtesy of Mathematical Reviews* S. D. Zeldin, USA

8. Churchill Eisenhart, Millard W. Hastay, and W. Allen Wallis, Techniques of statistical analysis, McGraw-Hill Book Co., New York, 1947, 473 pp. Cloth, 9.25 × 6 in., 45 figs., $6.

This book has been prepared by the Statistical Research Group, Division of War Research, Columbia University. It consists of three main parts: industrial statistics, planning experiments, and techniques and tables. The first part deals with acceptance inspection, quality control, tolerance limits for normal distributions and production problems of this kind. The second part is of special interest to the scientists and comprises some chapters about the planning of experiments for comparing two percentages or two standard deviations, and for estimating the mean and standard deviation of a normal distribution from observations on the cumulative distribution. Other chapters deal with the utilization of limited experimental facilities, when the cost of each measurement depends on its magnitude, and with experiments in

which the independent variable is known only in terms of probability distribution. Also the problems of seeking maxima and abandoning an experiment prior to completion are discussed. The third part gives techniques and tables for the probability that sample means are in opposite order to population means, for the significance of the largest of a set of sample estimates of variance, for inverse-sine transformation of proportions, and for elements of sequential analysis. W. Weibull, Sweden

9. **S. S. Wilks, Sampling and its uncertainties,** Proc. Amer. Soc. Test. Mat. 48, 859–876 (1948).

The author divides sampling into two stages. Stage I deals with variability of test results due to selecting only a small fraction of a lot to test; stage II deals with the variability of test results due to making only a small number of measurements on the material selected at stage I.

The purpose of the paper is to discuss the basic principles of sampling, and it deals primarily with mathematical and statistical aspects of sampling, rather than the experimental and practical aspects. It succeeds in giving a clear discussion of the precision (validity) and accuracy (reproducibility) of direct sampling. A. W. Rankin, USA

10. Eugen Bodea, Giorgi's rational MKS-system of units with consistent dimensions (Giorgi's rationales MKS-Mass System unit Dimensionskohärenz...), Verlag Birkhäuser Basel, 1949, 142 pp. Paper, 9.5 × 6.5 in., 17 tables, $5.

There are two kinds of problems associated with dimensions and units: the practical problems of usage, and the theoretical ones of the number and nature of the fundamental dimensions (the latter connect with dimensional analysis). To the practical problems the book makes a valiant contribution by giving a detailed picture of the grim chaos of holdovers by usage and tradition, and by explaining in much detail the Kalantaroff system of dimensions and the "coherently" associated Giorgi system of units. ("Coherence" means here that the relations between the units are identical with those between the dimensions. For instance, the set: dyne, gm, cm, sec is coherent with [L], [T], [M], because [F] = [M] [L] [T^{-2}].) The Kalantaroff system uses the following basic dimensions: length L, time T, electric charge Q, and magnetic flux Φ. H = QΦ is mechanical action, so that L, T, H yields a system of mechanical dimensions. The coherent system of units of Giorgi is: meter, second, coulomb and weber. Another coherent system of atomic units is also proposed. The practical advantages of the system are demonstrated with the aid of numerous, laborious and useful comparative tables. The latter constitute a major attraction of the book.

The theoretical contribution is negative, if any. The question of the number of basic dimensions has been settled in print too many times to require references: the number of basic dimensions is relative to the set of equations used in a particular theory, and may be reduced to zero if sufficiently many equations are used (thus, in the whole of physics all quantities can be made absolutely nondimensional by making the velocity of light, the constant of gravitation, and, say, the Planck constant all equal to unity). The author does not make this statement vigorously, but seems to be aware of this fact. And yet his discussion is confusing and sometimes downright contradictory (see footnote on page 53). He says almost explicitly that quantities "different in nature" cannot have the same dimensions: this is surely not a responsible statement. Be that as it may, the book is still interesting and useful, provided that all theoretical considerations are simply ignored. A. W. Wundheiler, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 25, 113)

11. E. A. Milne, Vectorial mechanics, Interscience Publishers, Inc., New York, 1948, 382 pp. Cloth, 9.7 × 6.2 in., 112 figs., $7.50.

The meaning of the title is obviously "Mechanics in vector symbolics." The book however consists of about half vector and tensor analysis and half mechanics in vector language and is divided into three parts: vector algebra (96 pages), systems of line vectors (including kinematics and statics, 82 pages), and dynamics (225 pages). The selection and arrangement of topics follows that in most texts of so-called advanced mechanics. Among books of the same kin, known to this reviewer, this one seems to him to be one of the best.

The author quotes Sydney Chapman as follows: "Vectors are not merely a pretty toy suitable only for elegant proofs of general theorems, but a powerful weapon of workaday mathematical investigation, both in research and in solving problems of the types set in English examinations."

If fifty years of continuous use of vectors have not yet established their power as a tool, there is little a book can do about it, especially if its ratio of application to theory is equal to 7/5. However, nobody denies the usefulness of vector calculus—the question is, how much of vector calculus is useful?

The tool value of any symbolics being hunch stimulation, a reasonable ratio must be maintained between the energy of memorization necessary to use the symbolics, and the amount of visualization obtained. Since neither of these is a measurable quantity, the critical value of this ratio is bound to remain a matter of opinion. This reviewer believes that the critical value is reached somewhere around the ternary vector operations (double vector product, dyadics etc.). Memorization at this point becomes too laborious to produce that effortless, instinctive handiness of the right formula at the right moment, so essential to good hunch stimulation. Moreover the "ternary" formulas in direct vector and dyadic symbolics are not a bit simpler than those in tensor (index) symbolics which has the tremendous advantage of managing with common (real-number) algebra, and not requiring any special operators and operations. Since the author gives an exposition of tensor, as well as of the vector calculus, his insistence on vector notation in involved formulas is difficult to understand.

There are many commendable features in the author's presentation of the vector calculus. The archaic and wrong "magnitude and direction" definition is not mentioned at all, although the purely algebraic definition is still not given. Sliding ("line") vectors are defined carefully, but no mention of the fact is made that systems of line vectors are represented by skew tensors; thus an occasion of demonstrating the usefulness of tensors and improving the theory/application ratio is missed. The ϵ tensor density (of components +1, −1, 0) is introduced with advantage (it is called a tensor, which is true only under transformations of determinant 1). A. W. Wundheiler, USA

12. Earle Buckingham, Analytical mechanics of gears, McGraw-Hill Book Co., New York, 1949, 546 pp. Cloth, 9.2 × 6.3 in., 244 figs., $10.

The author has been in touch with modern developments in gear problems through his active association with ASME gear committees. Many of the neat relationships deduced in this book were available only in magazine articles and in trade publications. It was not written, however, to serve as a textbook in gear design. Instead, it will serve as a handbook and a guide for machine designers and students. The algebraic and trigonometric relationships are derived from many clear drawings. As each problem is presented, the symbols are completely defined making it possible to follow the derivation without reference to other portions of the book.

The kinematics of conjugate gear-tooth action, which is the main theme of this book, includes the determination of the cycloidal and trochoidal paths traced by special points on the gears, the paths of tooth contact, the tooth forms and their radii of curvature, sliding action and methods of measuring and checking tooth forms. These tasks have been performed for parallel shafts coupled by spur gears, internal gears or helical gears; for intersecting shafts coupled by bevel gears; and for nonparallel, nonintersecting shafts coupled by spiral gears, hypoid gears, straight worm and wheel pairs or hour-glass worm and wheel pairs. Descriptions are given of special modifications of these, such as formate bevel gears and formate hypoid gears, lantern pinions, face gears, inclined worms and hobbed splines. These have been motivated by special methods of manufacture. Tables of coordinates of tooth forms and contact lines accompany their equations.

Formulas for efficiency and strength cannot be entirely analytical; they involve empirical constants which are tabulated for the various tooth forms. The resistance of the surface to wear also depends upon the materials used; plotted data are included for this determination. The factors involved in dynamic loading and limiting loads are discussed. Michael Goldberg, USA

13. I. I. Artobolevskiĭ and B. M. Abramov, On a form of the equations of motion of a machine (in Russian), Doklady Akad. Nauk SSSR 59, 1261–1264 (1948).

The angular velocity ω of the driving link of a rotating or vibrating mechanism varies with the following quantities: the moment of inertia J, and the difference ΔM between the driving moment and the resisting moment, each of which varies with the position angle ϕ. The motion is solved by a step-by-step integration process in which constant "fictitious" values (designated by the zero subscripts) are used over intervals in accordance with the reduced equation $\Delta M_0 = J_0 d\omega_0/dt$. The method is illustrated by a practical graphical example. M. Goldberg, USA

14. R. Buckley and E. V. Whitfield, Impulses and constraints in classical mechanics, Appl. sci. Res. Sec. A, 1, no. 4, 306–312 (1949).

The paper presents the basic theorems about impulsive constraints in the language of multidimensional geometry and matrix notation. Ed.

15. V. V. Dobronravov, On a relation in Euler's problem on the motion of a rigid body around a fixed point (in Russian), Doklady Akad. Nauk SSSR 65, 143–144 (Mar. 11, 1949).

This paper shows that, in the case of Euler's motion, the fixed point moves on a spherical curve, relative to the ellipsoid of gyration. Alexander Hrennikoff, Canada

16. Antonio Pignedoli, On the natural curves of a dynamic system (in Italian), Atti Sem. Mat. Fis. Univ. Modena 2, 37–59 (1948).

If V_n is a Riemannian space with the line element ds, and if $F(P)$ is a point function defined on V_n, the curves in V_n for which the value of the integral $\int F(P)ds$ is stationary are called natural curves. Various choices of the function $F(P)$ lead to various important natural curves. For instance, V_n may be the configuration space of a conservative holonomic dynamical system, and the natural curves may be the trajectories in this space, corresponding to natural motions of the system with a prescribed value of the energy.

In the first part of this paper the familiar generalities concerning natural curves, such as the defining differential equations, are developed by means of vector methods. Particular attention is given to the relations between the theory of natural curves and the Hamilton-Jacobi theory of dynamics, and also to certain equivalences between different problems leading to families of natural curves. In the latter parts of the paper the author gives some new theorems concerning the properties of natural curves.
Courtesy of Mathematical Reviews L. A. MacColl, USA

17. Ed. Batschelet, An exception to Poincaré's ergodic theorem (in German), Experientia 4, 270 (1948).

An elementary proof of the well-known fact that, in the billiard-ball problem on an elliptical table, a motion through a focus is asymptotic in both senses to the major axis.
Courtesy of Mathematical Reviews G. A. Hedlund, USA

18. V. V. Nemĭtskiĭ, On the theory of orbits of general dynamic systems (in Russian), Mat. Sborn. 23, 161–186 (Sept. 1948).

This paper contains proofs of results previously announced by the author [C. R. (Doklady) Acad. Sci. URSS (N. S.) 53, 491–494 1946].
Courtesy of Mathematical Reviews W. H. Gottschalk, USA

19. Richard Bellman, On the boundedness of solutions of nonlinear differential and difference equations, Trans. Amer. math. Soc. 62, 357–386 (Nov. 1947).

Let z denote a vector with N components and A a constant matrix. The author considers the system

$$dz/dt = (A + B_1(t))z + B_2(t)dz/dt + f(z, dz/dt, t),$$

where the norms of B_1 and B_2 are small and the first derivatives of $f(z, w, t)$ with respect to z_k and w_k tend to zero as $||z|| + ||w|| \to 0$, uniformly in t. The system where dz/dt does not appear on the right has been widely treated, and the author adopts a familiar integral-equation formulation together with a successive-approximation technique to treat the case where several or all of the characteristic roots of A have negative real parts. He shows that this technique can be used to study stability in the case where all solutions of $dy/dt = A_y$ are bounded as $t \to +\infty$. Here it is necessary for $\int_0^\infty ||B_i(t)||\, dt$ to be small and for the first derivatives of $f(z, w, t)$ with respect to z_k and w_k to satisfy a similar integrability condition. Using the Birkhoff-Kellogg fixed-point theorem the author proves corresponding results, some of them new, for $dz/dt = A_z + f(z, t)$ under weakened hypotheses of the type $||f(z,t)|| = o(||z||)$ as $||z|| \to 0$. He also shows how the methods can be used on difference equations.
Courtesy of Mathematical Reviews N. Levinson, USA

20. Cataldo Agostinelli, Nonholonomic systems with separated kinetic characteristics and the rolling motion of a heavy sphere on an arbitrary surface (in Italian), Atti Sem. Mat. Fis. Univ. Modena 2, 197–213 (1948).

A statement of conditions under which the equations of motion of a nonholonomic system, by introduction of suitable parameters, break up into two distinct systems of differential equations of the first order, each of which can be solved separately. Two illustrative examples are given: the rolling of a homogeneous and nonhomogeneous sphere on a given surface. In the first case, for example, by choosing as parameters the three components ω_1, ω_2, ω_3 of the angular velocity ω of the sphere, referred to the moving trihedral of which the first two axes coincide in direction with the tangents to the lines of curvature u = constant and v = constant of the surface S at the point of contact M with the sphere and the third axis lies in the direction of the normal N to the surface at the point M, the determination of motion of the

sphere is reduced to the integration of a system of five differential equations of the first order, involving the curvilinear coordinates u and v, which define the position of the point M on the surface S, and the three components $\omega_1, \omega_2, \omega_3$ of the angular velocity π of the sphere. A second system of equations, also of the first order, which connects the components $\omega_1, \omega_2, \omega_3$ with the Eulerian angles and their derivatives, determines as functions of the time t, after the first system is integrated, the three Eulerian angles, which define the orientation of the sphere. In the case of a homogeneous rolling sphere it is shown that the only surfaces for which the component of the angular velocity of the sphere along the normal to the surface at the point of contact is constant, are the plane and the sphere.

Courtesy of Mathematical Reviews E. Leimanis, Canada

21. **Antonio Pignedoli, On the existence of a linear integral in Lagrangian velocities for a nonholonomic system** (in Italian), Atti Sem. Mat. Fis. Univ. Modena 1, 50–58 (1947).

This paper deals with the following problem: Given a material system whose position at any time t can be specified by n Lagrangian coordinates $q_1, \ldots, q_n$ and whose motion is subject to m $(m < n)$ nonholonomic constraints dependent on the time t, under what conditions do the equations of motion of the system admit an integral linear in the Lagrangian velocities $\dot{q}_1, \ldots, \dot{q}_n$? An analogous problem has been treated by Agostinelli [Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. 80, 231–239 (1945)], when there are ignorable coordinates and the constraints are independent of the time t. By introducing linearly instead of the Lagrangian velocities $v = n - m$ parameters ω and by expressing the equations of motion in terms of quasi-coordinates, the conditions for existence of an integral of momentum with respect to one of the ω's are stated. A special case is discussed in which such an integral in fact exists which, expressed in terms of the Lagrangian velocities, necessitates the existence of an ignorable coordinate.

Courtesy of Mathematical Reviews E. Leimanis, Canada

22. **Antonio Pignedoli, On the applicability of Jacobi's method of analytical mechanics to nonholonomic systems** (in Italian), Atti Sem. Mat. Fis. Univ. Modena 1, 78–94 (1947); 2, 87–95 (1948).

The principal result of this paper is the following theorem. Consider a material system subject to bilateral constraints without friction and suppose that the position of the system at any instant is specified by n Lagrangian parameters. If in addition the motion of the system is subject to m $(m < n)$ nonholonomic constraints, the solution of the problem of motion can be reduced to the determination of a single function V, regarded as a function of the Lagrangian parameters and the time t and containing $n - m$ arbitrary constants, which satisfies a system of $m + 1$ partial differential equations of the first order, and to the integration of a system of m ordinary differential equations of the first order in m unknown functions, the latter system being derived from the equations of nonholonomic constraints. Under somewhat less general assumptions the problem has been recently considered by Dobronravov [C. R. (Doklady) Acad. Sci. URSS (N.S.) 22, 477–480 (1939)], and Lampariello [Atti Accad. Italia. Rend. Cl. Sci. Fis. Mat. Nat. (7) 3, 734–740 (1942); 4, 12–19, 20–28 (1943)]. In the second paper the author states necessary and sufficient conditions for the consistency of the system of $m + 1$ partial differential equations of the first order for the function V, and gives the characterization of nonholonomic systems to which the Jacobi-Hamilton method of integration is applicable.

Courtesy of Mathematical Reviews E. Leimanis, Canada

23. **Antonio Pignedoli, Particular solutions of a nonholonomic system on the basis of existence of integrals or invariant solutions** (in Italian), Atti Sem. Mat. Fis. Univ. Modena 1, 95–118 (1947).

The principal theorem of this paper may be regarded as an extension of a well-known theorem of Levi-Civita [Atti Accad. Lincei, Rend. Cl. Sci. Fis. Mat. Nat. (5) 10, 3–9 (1901); cf. also T. Levi-Civita and U. Amaldi, *Lezioni di Meccanica razionale*, v. 2, part 2, Zanichelli, Bologna, 1927, chapter 10, §9; and E. T. Whittaker, *A treatise on the analytical dynamics of particles and rigid bodies*, 3d ed., Cambridge University Press, 1927, p. 325], which establishes a connection between the invariant relations or integrals of a holonomic system and certain families of particular solutions of the equations of motion, to nonholonomic dynamical systems. The theorem may be stated as follows: Consider a material system whose position at any time is specified by n Lagrangian parameters and whose motion is subject to m $(m < n)$ nonholonomic constraints independent of the time. Suppose that the equations of motion, by introduction of $\nu = n - m$ suitable parameters ω, are written in the canonical form. Then to any set of l $(l < \nu)$ invariant relations (or integrals) of the canonical system, which are in involution, if the nonholonomic constraints are such that certain further conditions are satisfied, there corresponds a family of ∞^{m+l} (or ∞^{m+2l}) particular solutions of the canonical system, whose determination depends on the integration of a system of $m + l$ ordinary differential equations of the first order. The theorem is applied to the special case in which the possible motions may be regarded as an extension to nonholonomic systems of those motions which in the case of holonomic systems are known as "motions à la Routh."

Courtesy of Mathematical Reviews E. Leimanis, Canada

Gyroscopics, Governors, Servos

(See also Revs. 15, 46, 140)

24. **A. Porter, Basic principles of automatic control systems,** Proc. Instn. mech. Engrs. 159, no. 38, 25–45 (1948).

This is an expository paper. Martin Goland, USA

25. **A. M. Letov, The regulation of the stationary state of a system subjected to constant perturbing forces** (in Russian), Prikl. Mat. Mekh. 12, 149–156 (Mar.-Apr. 1948).

A certain mechanical system with a regulator gives rise to the following system:

$$T^2\ddot{\varphi} + U\dot{\varphi} + k\varphi + \mu = \chi, \ \dot{\varphi} - \dot{\beta} - n\beta = -n\epsilon,$$
$$\dot{\mu} + \rho\mu = F(\sigma), \ \sigma = a\varphi + E\dot{\varphi} + G^2\ddot{\varphi} - l^{-1}\mu,$$

where μ is the displacement of the regulator, σ is the displacement of the starting device, φ and β are the angular displacements of the mechanism from zero (the desired position). In addition T, U, k, n are constants of the basic mechanism; a, E, G, l are constants of the regulator; χ and ϵ are constant perturbing forces; ρ is a constant of the servomotor such that in the first approximation $\rho\mu$ is to compensate for the load on the servomotor. Finally $F(\sigma)$ is a known characteristic function which is continuous and of the same sign as σ. A stationary regime (all derivatives zero) exists whose stability both in the large and in the small is investigated by Liapounoff's second method [A. Liapounoff, *Problème Général de la Stabilité du Mouvement*, reprinted as Ann. of Math. Studies 1947, no. 17] and in particular utilizing a theorem of A. I. Lurye [Appl. Math. Mech. (Prikl. Mat. Mekh.) 9, 353–367 (1945)]. The case where $\rho = 0$ (servomotor independent of load) is also discussed by Liapounoff's first method. Self-oscillations are treated following a linear approxi-

mation method due to B. V. Bulgakov [Appl. Math. Mech. (Prikl. Mat. Mekh.) 10, 313–332 (1946)] which consists in replacing $F(\sigma)$ by a linear approximation $h\sigma$ and looking for periodic solutions of the resulting linear system. Necessary and sufficient conditions for stability are discussed for the two cases of hard back coupling ($G = 0, l \neq \infty$) and no back coupling ($G \neq 0, l = \infty$).

S. Lefschetz, USA

26. Sudhansu Datta Majumdar and Gopendra Nath Sarkar, Speed control for the air driven ultracentrifuge, Rev. sci. Instrum. 19, 541–544 (Sept. 1948).

A very sensitive method of speed control, applicable to large angular velocities, is described. It uses the stroboscopic effect. By means of mirrors clamped to an electrically maintained tuning fork, light flashes of accurate frequency are sent to a mirror attached to the shaft of the controlled machine. The normal of this mirror is inclined at a small angle to the axis of rotation. If the speed of the machine is an integral multiple of the frequency of the flashes, the latter are reflected to the same spot of the screen. In case of speed deviation the light spot moves in a circle. By means of three mirrors placed near to each other, the light spot is reflected to three photoelectric cells which, through an amplifier, operate the correcting element according to the direction of motion of the light spot.

The arrangement was used with an air-driven ultracentrifuge of rotor speed 1024 rps, and kept the speed constant within the limits of 0.2 rps. If the motion of the light spot is controlled by using more sets of mirrors, an accuracy of 0.02 rps and more can be obtained. The description is very clear and detailed.

Miroslav Nechleba, Czechoslovakia

27. Guiseppe Peretti, On a curious gyroscopic effect (in Italian), Atti Sem. Mat. Fis. Univ. Modena 2, 75–86 (1948).

This note gives the theory of a certain toy, consisting essentially of a gyroscope mounted in an egg-shaped case. If the gyroscope is not rotating, the system is in stable equilibrium when the case rests on a horizontal plane with the large end down. However, if the gyroscope is set into rotation, the case turns over, and comes to a stable equilibrium with the small end down. It is shown that this behavior depends upon the presence of friction, and upon the parameters involved satisfying certain inequalities.

Courtesy of Mathematical Reviews L. A. MacColl, USA

Vibrations, Balancing

(See also Revs. 43, 46, 150, 186, 192)

28. Britton Chance, Frederick C. Williams, Vernon Hughes, Edward F. MacNichol and David Sayre, Waveforms, McGraw-Hill Book Co., New York, 1949, 785 pp. Cloth, 9.2 × 6.5 in., 749 figs., $10.

The object of *Waveforms* is to provide circuit designers with a reference book containing techniques of established value in the application of circuit design to the generation of waveforms and to the manipulation of waveforms to meet specific needs. The material of the book is classified according to circuit functions, in order that the circuit designer may conveniently find a compilation of useful circuits accomplishing a particular function.

The book begins with a survey of the properties of linear and nonlinear circuit elements which are applicable to all of the circuits in the book. This comprises the first three chapters. Commencing with chapter 4 the discussion proceeds according to the function of the circuit. First the generation of simple waveforms of both voltage and current such as sinusoidal and triangular, is followed by a discussion of methods of generating special waveforms such as trapezoids, exponentials, hyperbolas, parabolas, and curves approximated by segments. Special emphasis is given to methods which ensure their rapid initiation and the accurate control of their shape. Two chapters are devoted to a discussion of the methods for analyzing a waveform with respect to amplitude or time, and include the various methods of selecting a portion of a given wave. Special emphasis is placed upon the process of marking the instant of equality of two waves.

Variation of the parameters of amplitude or time-analysis methods also leads to important modulation and demodulation processes, and a treatment is given of those methods and demodulation which have been found to be of most use in precision circuits. The important process of linearly modulating the time interval between two portions of a waveform is discussed in some detail since this is a basic process in accurate time measurement, and a summary of methods is included.

Three chapters are devoted to the basic aspects of the synchronization of waveform generators to accomplish frequency multiplication and division or pulses. Many of these processes depend upon periodic counting circuits, and a chapter is devoted to their characteristics. A chapter treats the techniques used in the measurements of waveforms which can be carried out with a simple oscilloscope. Some discussion is given to delay devices which are a specialized class of components in which an input wave is reproduced at the output after a fixed delay.

Although the methods described in the book have been developed for application to radar apparatus and similar electronic devices, the contents may be of considerable value to the research worker in experimental and theoretical applied mechanics dealing with mechanical or electromechanical systems. Research in transients in mechanical systems often requires experimental techniques in which forces of special time history are applied to the system. *Waveforms* should be of considerable value in developing electromechanical devices for this purpose. The book should be valuable in developing analog computing machines using electrical circuits to replace mechanical systems. The techniques outlined for producing and manipulating waveforms could be applied in an interesting manner to analog computing machines for aeroelastic systems. Finally the theoretical research worker in applied mechanics can undoubtedly benefit from techniques in this book which, by analogy, may be applied to mechanical systems.

R. L. Bisplinghoff, USA

29. J. M. Burgers, Spectral analysis of an irregular function (in English), Proc. Kon. Ned. Akad. Wet. 51, 1073–1076 (Nov. 1948); 51, 1222–1231 (Dec. 1948).

An analytical study is presented of an irregularly oscillating function of time. An electrical signal, proportional to the irregular function, is passed through a filtering circuit. The filter produces an approximately harmonic oscillation out of the irregular function in such a way that the amplitude of this oscillation depends on a certain Fourier transform. A study is also given of the fluctuations which may be expected to be shown by the indication of a galvanometer, connected to a thermocouple through which the signal transmitted by the filtering circuit is passed.

R. C. Binder, USA

30. Henry E. Fettis, The calculation of coupled modes of vibration by the Stodola method, J. aero. Sci. 16, 259–271 (May 1949).

In his summary the author states that there are at present only two methods in common use for finding the natural modes of coupled bending-torsion vibration of beams: the Rayleigh-Ritz method, and the matrix iteration method. In the opinion of this reviewer the particular adaptation of the Stodola method which

is ably developed by the author in the present paper is superior to these other two methods. However, it might be pointed out that an even better method, which may be called the forced vibration method, has been in common use for about five years. This method was first published in the reviewer's *Vibration Analysis*, McGraw-Hill Book Co., 1944; it was also published in the Trans. Amer. Soc. mech. Engrs., Jan. 1945, under the title: "New method of calculating natural modes of coupled bending-torsion vibration of beams." N. O. Myklestad, USA

31. **R. A. Anderson and J. C. Houbolt, Effect of shear lag on bending vibration of box beams,** Nat. adv. Comm. Aero. tech. Note no. 1583, 15 pp. (May 1948).

Shear lag has been blamed as the cause of discrepancies that have been sometimes observed between the computed and the measured natural frequencies of aircraft structures. In this report, the effects due to shear lag are computed for two simple uniform box beams without cutouts, one of which (Beam A) was designed to have only a small amount of shear lag (calculated tip deflection under concentrated load increased by 2% due to shear-lag effects) and the other (Beam B) designed to have a very large shear lag (calculated tip deflection under the concentrated load increased by 19%). The analysis makes use of an "effective" stiffness which is determined by modifying the geometrical stiffness to account for the effects of shear lag by a method based on previous work carried out by Kuhn at the NACA. This effective stiffness is then used to compute the uncoupled natural frequencies of the two beams in bending by the Stodola method.

The results show that for Beam A, the fundamental frequency is reduced by only 2% due to the shear-lag effects, but the frequencies in the second and third modes are reduced by 12% and 22% respectively. The deflection shapes of all three vibration modes are only slightly affected by the shear lag and do not reveal by simple inspection the significant curvature changes responsible for these differences. In the case of Beam B, the fundamental frequency was reduced by 21%. Benjamin Smilg, USA

32. **O. Föppl, Critical speed of a rotating shaft** (in German), Schweiz. Bauztg. 65, 259–262 (May 1947).

A slender shaft carrying an axially symmetric rotor of appreciable diametral and axial moments of inertia generally exhibits two critical speeds. One of these becomes infinite for relatively narrow rotors, having a greater axial than diametral moment of inertia. However, for wide turbine wheels which have great diametral moments of inertia, there are two distinct critical speeds above and below the natural bending frequency corresponding to the equivalent concentrated-mass system. Two critical speeds also exist for modes in which whirling takes place in the direction opposite to the shaft rotation; ordinarily, these modes are highly damped and therefore unimportant. The author discusses each of these cases for symmetric and unsymmetric arrangements, giving formulas and graphs for calculating the critical speeds and natural frequencies. (See also Rev. **1444,** 1948.) G. A. Nothmann, USA

33. **Ya. S. Uflyand, The propagation of waves in the transverse vibration of bars and plates** (in Russian), Prikl. Mat. Mekh. 12, 287–300 (May-June 1948).

When the effects of shearing force and rotational inertia are taken into account, the equation (1) $\partial^4 u/\partial x^4 + c\partial^2 u/\partial t^2 = 0$ for the lateral vibration of a prismatic bar becomes

$$(2)\quad \frac{\partial^4 u}{\partial x^4} - \left(\frac{1}{{c_1}^2} + \frac{1}{{c_2}^2}\right)\frac{\partial^4 u}{\partial x^2 \partial t^2} + \frac{1}{{c_1}^2{c_2}^2}\frac{\partial^4 u}{\partial t^4} + c\,\frac{\partial^2 u}{\partial t^2} = 0.$$

This equation is solved by using a Laplace transformation. The solution consists of two waves propagated along the bar with velocities c_1 and c_2. The author considers a semi-infinite bar, suddenly loaded at the end, and a finite hinged bar suddenly loaded at the midpoint, with a concentrated constant force. For large values of the time t, the solutions of (2) converge to the corresponding equilibrium solutions of (1). The transient state, however, can be found only from (2). The same effects are considered for the lateral vibrations of a plate, and analogous results are found.

Courtesy of Mathematical Reviews W. H. Müller, USA

34. **G. Hok, Thickness-shear vibrations of thin anisotropic plates,** J. acoust. Soc. Amer. 20, 406–417 (July 1948).

The analysis in the present paper was occasioned by the extensive use of thickness-shear quartz plates for control of the operating frequency of radio transmitters and receivers. The aim of the paper is to derive the standing wave configurations from plane waves reflected from the boundaries of the plate, and to include the variation of amplitude in the third dimension. Only rectangular plates with one edge parallel to the crystallographic x-axis are considered. The medium is assumed to be conservative and the influence of piezoelectricity on the elastic constants is neglected. The fundamental differential equations are derived and their solution given first for an infinite medium. Next, the boundary conditions resulting from the boundary planes are introduced, and the effects of the incident and reflected waves in combining to a system of standing waves are discussed. The solution is particularly valuable for the qualitative interpretation it gives.

R. L. Bisplinghoff, USA

Wave Motion, Impact

(See also Rev. 173)

35. **Sokrates Rombakis, Phase velocity of waves of finite amplitude,** Z. Met. 1, 302–303 (July 1947).

This short note shows that the phase velocity of water waves of finite amplitude is given approximately by $(g\lambda/2\pi)/(1 - 4\pi^2A^2/\lambda^2)$, the difference between this and Stokes' value being of the second order in the amplitude/wave-length ratio. Ed.

Elasticity Theory

(See also Revs. 47, 56)

36. **Jean Ferrandon, On the rotational character of a deformation without cavitation in an elastic continuum, produced by a small displacement of a rigid core** (in French), Houille blanche 3, 445–446 (Sept.-Oct. 1948).

The author notes that if a rigid inclusion embedded in an elastic continuum is displaced, the deformation is necessarily rotational. He had essentially proved this in a previous paper [C. R. Acad. Sci. Paris, 226, p. 2047 (1948)] by assuming the existence of a displacement potential and meeting a contradiction.

G. F. Carrier, USA

37. **F. Reutter, Semiplane and parallel strip with variable elastic modulus,** (in German), Ingen.-Arch. 16, 307–320 (1948).

The stress distributions in a semi-infinite body with surface $x = 0$, and in an infinite plate with surfaces $x = \pm b$, are considered, when the elastic modulus E is a function of x. A body force in the x direction, depending on x, is included. Such solutions are of interest in soil mechanics, and in the theory of sandwich plates which are stronger if the lightweight filler between the metal faces is more rigid close to the faces. A sinusoidal dis-

tribution of normal surface stress $P_0 \sin my$ with only normal displacements is considered, this problem being suggested in connection with the instability of sandwich plates. In the case of the plate, both symmetric and antisymmetric loading on the two faces are treated. Solutions are obtained for quadratic, cosh, and exponential variations of E, some solutions being expressible in closed form, others as infinite series. Most of the solutions assume Poisson's ratio $\nu = 0$, but it is shown, by considering a special case, that this has comparatively little influence on the stress distribution. E. H. Lee, USA

38. H. Borowicka, Stress distribution in a half plane with a modulus of elasticity diminishing with the depth (in German), Öst. Ingen.-Arch. 2, no. 5, 360–363 (Sept. 1948).

A simple solution in closed form is given for the plane stress distribution in an elastic half plane due to a concentrated normal force on the boundary, assuming Young's modulus to be inversely proportional to the distance from the boundary. By integration the solution is found for a normal load uniformly distributed over a segment of the boundary. This follows the author's earlier paper [Ingen.-Arch. 14, 75 (1943)]. H. J. Greenberg, USA

39. F. Szelagowski, Stress distribution in a disk with rigid core when submitted to tension or compression in its plane (in French with German and English summaries), Publ. int. Ass. Bridge Struct. Engng. 8, 271–279 (1947).

The problem the author deals with has already been solved exactly in a more general way by L. H. Donnell, "Stress concentrations due to elliptical discontinuities in plates under edge forces," *Theodore von Kármán Anniversary Volume*, 1941, p. 293. J. J. Koch, Holland

40. Mauro Picone, On the torsion of an elastic hollow prism after Saint-Venant's theory (in Italian), Pubbl. Ist. appl. Calcolo, no. 222, 347–372 (1948).

The author considers questions of regularity of the solution to the Neumann problem which arises in connection with the pure Saint-Venant torsion of a hollow prism of cross section bounded by two concentric rectangles, parallel but not necessarily similar. Ed.

Experimental Stress Analysis

41. Wilfred Heller and Hans Oppenheimer, Comparative studies on photoelasticity of elastomers and plastomers, J. colloid Sci. 3, 33–34 (Feb. 1948).

Experiments are described wherein thin films (generally 180–250 μ thick) of polythene, naturally cured rubber and GRS were strained and then studied for stress double refraction. Curves showing final double refraction (after changes with time had gone to practical completion) against elongation show a nearly linear increase in birefringence with elongation up to rupture for non-crystallizing elastomers such as GRS. Crystallizing polymers, such as natural rubber, show a sharp increase in birefringence on elongation within the crystallizing range whereas plastomers, such as polythene, show a very steep increase in double refraction at low elongation followed by an approach to saturation at moderate elongations. Further tests in which the strips had been stressed to a predetermined amount and held showed that polythene was unaffected by time, GRS and natural rubber at low elongations showed a slight decay, but at high elongations the double refraction in natural rubber increased with time.

Studies were made of the residual permanent set and residual permanent double refraction in polythene from which it is concluded that polythene is predominantly elastic at low elongations, but at higher elongations a high degree of irreversible orientation of polymer chains is induced. Further preliminary studies of polythene suggest a correlation between the magnitude of restoring force and the rate of optical relaxation which follows a rapid "partial" retraction.

The work in general, while stated to be of a preliminary nature only, serves to emphasize the value of combining photoelastic with rheological studies of high polymers. The method offers promise as a practical testing technique for polymers. F. T. Barwell, Scotland

42. K. H. Swainger and J. Twyman, An optical rectangular rosette extensometer for large strains, J. sci. Instrum. 25, 187–189 (June 1948).

This instrument was designed for determining the stress-strain relationship in post-yield straining of metals. It permits measurement of strains in two orthogonal directions and a third at 45 deg to the other two, with large strain ranges and high sensitivity.

The three gages used are mirror-extensometers. The extensometer body is furnished with a conical leg, which is the common fixed point of the gages, and with three movable legs, each of which operates a lozenge and mirror. Knife-edge fittings permit setting the instrument back to zero when its effective range is covered, and so give a large operating range.

Gage lengths between about 1 and 4 in. are available. The sensitivity is 2.5×10^{-5} in. per unit strain. The scale of 100 cm. covered without resetting the instruments, corresponds to a strain of 0.125/in. gage length. Giulio Ceradini, Italy

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 33, 53, 60, 67)

Plates, Disks, Shells, Membranes

(See also Revs. 34, 37, 47, 54, 67)

43. Aldo Ghizzetti, Application of Ritz's method to the calculation of the fundamental frequency of a circular membrane with a circular eccentric hole (in Italian), R. C. Accad. Lincei, ser. 8, 2, 559–564 (May 1947).

The author investigates the fundamental frequency of a circular membrane (circumference C_0, radius 1) with a circular eccentric hole (eccentricity s, circumference C_1, radius r), namely the lowest characteristic value κ_0 of the problem $\Delta_2 u + \kappa^2 u = 0$, with the boundary conditions $u = 0$ (along C_0), $\partial u/\partial n = 0$ (along C_1).

The transformation of Möbius changes C_0 and C_1 in the concentric circumferences Γ_0 (radius 1) and Γ_1 [radius $q = q(r,s)$], while the differential equation becomes (in polar coordinates) $\Delta_2 u + \sigma^2 f(\rho,\vartheta) u = 0$ ($u = 0$, along Γ_0; $\partial u/\partial n = 0$, along Γ_1) (1).

According to Ritz's method $\sigma_0 = \min \mu$, where the functional $\mu = I[u]/H[u]$ is calculated in the class of functions u which satisfy the boundary conditions; in this case

$$I[u] = \int_q^1 \rho d\rho \int_0^{2\pi} \left[\left(\frac{\partial u}{\partial \rho} \right)^2 + \frac{1}{\rho^2} \left(\frac{\partial u}{\partial \vartheta} \right)^2 \right] d\vartheta,$$

$$H[u] = \int_q^1 \rho d\rho \int_0^{2\pi} f(\rho, \vartheta) u^2 d\vartheta$$

M. Picone suggested to substitute the function u by a linear combination ${}_n\Sigma_0^N \gamma_n^{(N)} v_k$, where v are the solutions of problem (1); in the case of no eccentricity [$f(\rho,\vartheta) = 1$] and $\gamma_n^{(N)}$ are, for every

N, the constants which minimize the functional μ_N. (When $f = 1$, the solutions v are known).

The author proves that the method really converges, namely, $\lim \mu_N = \lim I[\Sigma_0^N \gamma_n^{(N)} v_n]/H[\Sigma_0^N \gamma_n^{(N)} v_n] = \sigma_0$ for $N \to \infty$.

He then calculates nine values of κ_0, approximate in excess, corresponding to different values of r and s.

D. Gentiloni-Silverj, Italy

44. **P. Cicala, The circular shell with equal panels and stringers under transversal loads** (in Italian), Monogr. sci. Aero. no. 7, 3–18 (Aug. 1947).

As is usual in airplane fuselages, the shell consists of a cylindrical skin, equal stringers at equal intervals, and circular rings. Loads are applied at the intersections of stringers and rings only. The skin acts only as a shear web. The solution of the stress problem is obtained by representing loads and stresses as finite Fourier sums. The general problem is the same as in the paper by E. Ebner and H. Köller [Luftfahrtforsch. München 14, 607 (1937)]. There the emphasis is on the end-constraint problem, while the author's method seems particularly fit for local concentrated loads applied somewhere to the shell.

W. Flügge, USA

45. **P. P. Bijlaard, Approximative method of analysis for rectangular reinforced concrete plates under uniformly distributed or hydrostatic load** (summaries in French and German), Publ. int. Ass. Bridge Struct. Engng. 1948, 507–517 (prelim. publ).

The author considers the method he developed in previous papers [Ingenieur 49, B88-B93 (1934); 50, Bt.37-Bt.52 (1935)] for the approximate determination of the maximum bending moments in rectangular plates, any edge being either clamped or freely supported. The method is based on an analysis of the plate as a system of beams at right angles and leads to simple formulas for these moments; the results furnished are in good agreement with those furnished by more analytical methods. Both uniform and hydrostatic loads are treated and the application here is to plates of reinforced concrete wherein Poisson's ratio is taken to be zero. The method also furnishes the values of maximum clamping moments and hence of carry-over factors necessary for analysis of continuous floor slabs by the Cross method.

H. J. Greenberg, USA

46. **R. B. Green, Gyroscopic effects on the critical speeds of flexible rotors,** J. appl. Mech. 15, 369–376 (Dec. 1948).

In the calculation of critical speeds of flexible rotors the gyroscopic effect is frequently considered only for forward synchronous precession. While this consideration is sufficient to cover most practical systems, there are numerous cases in which the gyroscopic effects due to nonsynchronous precession are evident. Possible excitations of this kind include the Soderberg and Stodola secondary criticals, internal hysteresis, oil-whip friction in the bearings and vibrations from without the system.

In this article, the forces and moments on rotating disks are found for the general case, and the various possible criticals of common rotor systems are determined and plotted for all degrees of gyroscopic effect. Specific examples are shown of a cantilever system, two simply supported single-disk systems, a double-disk system, and an infinite-disk system. Methods of solution are indicated for any case. The gyroscopic effect is shown for all possible ratios of shaft-rotating frequency to whirl frequency.

E. G. Allen, USA

Buckling Problems

(See also Revs. 53, 54)

47. **C. F. Kollbrunner and G. Herrmann, Stability of plates in the plastic range** (in German), Mitt. Inst. Baustat. Eidgenöss. tech. Hoch. no. 20, 81 pp. (1947).

The authors refer to a previous report (no. 17, by Kollbrunner, in the same series of publications) on tests on buckling of rectangular plates in the elastic and plastic range of strains as part of a program of investigations which was carried out for the Engineering Commission of the Association of Swiss Bridge and Steel Construction Builders. In the present report (no. 20) the authors give a free German translation of a 1944 paper by A. Ilyushin on the theory of the stability of thin plates and shells beyond the elastic range, and include some of their own previous work. The authors attribute a great merit to Ilyushin's theory of buckling, and state that they endeavored to express certain general conclusions of this theory in a form which they believe would be more suitable in engineering applications. Forty-four pages are devoted to the derivation of the buckling formulas, and 25 pages to the development of curves in which a representative buckling or equivalent stress is plotted as a function of a slenderness-ratio for certain cases of rectangular plates loaded by forces acting in the direction of the middle plane of the plates (the original paper by Ilyushin had 23 pages). Due to an elaborate system of abbreviations for a number of parameters and quantities which the Russian author introduced in his analysis, it is not possible to discuss the formulas in this review. A. Nádái, USA

48. **Herbert Becker, The optimum proportions of a long unstiffened circular cylinder in pure bending,** J. aero. Sci. 15, 616–624 (Oct. 1948).

The paper gives an analytical method to determine the dimensions of a minimum-weight unstiffened circular cylinder under bending. It is assumed that the buckling stress σ of the cylinder may be expressed in the elastic range in the form $\sigma = KE(t/R)$ where t is the thickness, R the radius, E Young's modulus and K a constant. In the plastic range the author examines the minimum weight conditions resulting from a substitution of either the secant, reduced, or tangent modulus for E in the buckling equation. Using a three-parameter expression for the stress-strain curve he proves that by either choice approximately the same optimum design dimensions are obtained. The author states that substantial, but not conclusive, experimental evidence exists for the secant modulus assumption. The paper concludes with a simplified design procedure which avoids the use of a stress-strain relation and is based on the secant-modulus theory.

J. Hadji-Argyris, England

49. **S. Dei Poli, On the elastic stability of a cylindrical strip compressed along the generators** (in Italian), Atti Accad. Torino, 81–82, no. 1, 95–101 (1945–1947, issued in 1948).

The paper investigates the stability of a cylindrical panel compressed along the generators and supported along the edges by stringers. The stringers are assumed perfectly rigid against stretching and of negligible bending rigidity. Large deflections (of the order of the thickness) are taken into account by the use of two partial differential equations of equilibrium and compatibility previously established by P. Cicala. The stress function and the radial displacements are assumed in terms of finite trigonometric series which satisfy the boundary conditions. The unknown coefficients of the terms of this series are adjusted by means of Galerkin's procedure so as to satisfy (within the approximation afforded by the assumed finite series) the governing differential equations. The result is a set of two simultaneous

algebraic equations which define implicitly a relation between the load and the deflection. A load vs. deflection plot is a curve with three branches starting with a horizontal tangent (except in one case in which two branches start with different tangents from the same point).

Curves are presented for several numerical values of the geometrical parameters involved; they show that in many cases the load decreases as the deflection increases, the amount of decrease depending on the dimensions of the panel. It is also shown how the error in the linearized theory increases with the curvature.

Bruno A. Boley, USA

50. Chi-Teh Wang, Inelastic column theories and an analysis of experimental observations, J. aero. Sci. 15, 283–292 (May 1948).

Joints and Joining Methods

51. M. Ros, Fatigue strength of joints (in French), Eidgenöss. MatPrüfAnst. Ber. no. 161, 28 pp. (1948).

The fatigue strength of welded joints is analyzed by means of theoretical investigation and experimental work. The theoretical investigation is based on the classical theories of strength and most used of them is the maximum-strain theory which is applied to isotropic and quasi-isotropic materials. Welded joints are compared with riveted joints. The experimental work is rather comprehensive and deals with many types of welded joints in different heat-treatments. The primary material and the welded pieces are tested on macro- and microstructure, Brinell hardness, static tensile strength, Charpy impact resistance, bending ductility and fatigue strength in pulsating tension. The results are described in tables, diagrams and many illustrations, which show the different ruptures. Ragnar Nilson, Sweden

52. E. C. Hartmann, Marshall Holt and A. N. Zamboky, Static and fatigue tests of arc-welded aluminum alloy 61S-T plate, Welding Res. Supplement 22, no. 3, 129–138 (March 1947).

Static and fatigue tests are described of 15 types of specimens having butt joints, single-strap or double-strap joints with longitudinal or transverse fillet welds, scalloped joints, or welded-on attachments with no joint in the main plate. The main purpose of the paper is to show the relative effects of eccentricities and discontinuities in welding higher-strength aluminum alloys. The tests were confined to a stress range from zero to a maximum stress in tension. The frequency was 210 cpm and a fatigue test was considered finished at the formation of a visible crack or at the end of about 25,000,000 cycles.

The static strength was reduced approximately 50% by the heat of welding, which means that no complete annealing occurs during welding.

The welded joint with the greatest fatigue strength was a simple butt joint with the weld bead dressed flush, the fatigue strengths ranging from 49% to 54% of those of the unwelded plate. Further test results are discussed in some detail.

Finally, a few results of tests on specimens welded by the argon-shielded tungsten-arc method are communicated. These showed a definite improvement in fatigue strength, which is probably due to smoother welds.

F. J. Platema, Holland

Structures

(See also Revs. 44, 50, 80, 178)

53. Leopold Herzka, Statics of the deformations of steel plate girders, Springer-Verlag, Vienna, 1948, 232 pp. Paper, 9.6 × 6.75 in., 122 figs., $12.

The author applies the method of elastic weights to the analysis of members of variable section with arbitrary loads. He observes that the elastic weights of any segment can be replaced by the simple beam reactions at the ends of the segment without affecting the deflections at the ends of the segment. Then he obtains the solutions for a number of problems by simple combinations of basic elements. The techniques of solution are applied to many problems, including deflection of girders with cover plates, deflection of Vierendeel trusses, bending of masts and of frames, buckling of columns, etc.

Probably American readers would use for such problems the methods of column analogy and moment distribution developed by Hardy Cross, or one of several numerical procedures.

N. M. Newmark, USA

54. W. H. Wittrick, Preliminary analysis of a highly swept cylindrical tube under torsion and bending, Austral. Counc. Aero. Rep. no. 39, 23 pp. (May 1948).

The author presents an analysis of the stress distribution in highly sweptback cylindrical tubes with rigid diaphragms parallel to the main airflow. The method is essentially an extension of the theory developed by the reviewer and Dunne for unswept or slightly swept conical or cylindrical tubes (J. roy. aero. Soc. 51, 199–269, 757–784, 884–930 (1947); 53, 461–483, 558–620 (1949); see Rev. 626, Apr. 1948). The paper is subdivided into four main sections. In the first, the author, after developing the stress and strain relations and the compatibility equations, obtains the differential equation for a stress function F. Also given are expressions for the deflections, twist, and warping in terms of the stress distribution. The second section proves that if the thicknesses vary lengthwise exponentially, the stress function F may be expressed in the form (1) $F = h(s)\exp(\lambda z)$ where λ is a constant, z the distance along the generators, and $h(s)$ a function depending solely on the dimensions of a cross section. Moreover, the ribs rotate about points on a straight axis parallel to the generators. The differential equation for h is derived and it is shown that the stresses and stress-resultants vary as $\exp(\lambda z)$, the resultant normal to the ribs being zero. In order to deal with prescribed end conditions it is necessary to obtain self-equilibrating stress systems; these are investigated by the author in the third section. Each self-equilibrating stress system with a stress function of the type (1) has a distinct value of λ and a warping proportional to the direct stress. The λ_i's are the roots of a characteristic equation, the corresponding h_i's being the eigenfunctions. The eigenfunctions are not orthogonal, as in the unswept case, and this limits considerably their value for practical applications. In the fourth and last section the author obtains stress systems which remain constant along the length of the tube. For an unswept tube these stress systems are found to reduce to those of pure bending and Bredt-Batho torque. Finally the author derives the direction of the "zero-curvature axis" which is the axis about which the resulting couple must be applied so that the ribs rotate in their plane without curvature.

John Hadji-Argyris, England

55. N. J. Hoff, Vito L. Salerno, and Bruno A. Boley, Shear stress concentration and moment reduction factors for reinforced monocoque cylinders subjected to concentrated radial loads, J. aero. Sci. 16, 277–288 (May 1949).

Results of an investigation of the interaction between reinforced circular monocoque cylinders and the reinforcing rings are presented in the form of charts giving the shear in the skin and the

bending moments in the ring for the case of a single radial load applied to the ring. The theoretical basis for these charts is not given in detail, but it is indicated that deformations of the cross sections of the shell and warping of the cross sections have been taken into account. The charts should be of particular interest to aircraft stress analysts. A brief discussion of the related work of other investigators is also given. John E. Goldberg, USA

56. **L. D. Child, A study of a slab action in concrete pavements under static loads,** Proc. Highw. Res. Bd., 27 Ann. Meet., 64–84 (1947).

In 1944 the University and the Highway Department of the State of Michigan began a program of research into the effect of axle load upon concrete slabs. Preliminary tests were made upon a small model; in order to determine absolute values, it was desirable to repeat the experiments upon a full-scale slab. Loads were applied by means of one, two or three truck axles, at varying axle distances with dual 10.00–20 tires at 70 psi air pressure. The condition of the subgrade was dry and saturated, the subgrade modulus k was found by two different methods to be 110 and 60 lb per cu in. for the dry and saturated condition, respectively. Loadings were applied in the interior, at the free edges, at the joints and at the corners of the slab.

(a) The stresses measured under single loads, when compared with those computed from Westergaard's and Arlington's formulas, showed that the measured stresses exceeded the stresses computed from either formula for any load: corner, longitudinal edge, transverse edge or interior. The difference was considerable (30–40%); the Arlington formula gave closer values. (b) Maximum stresses were produced by a single axle on the corner of the slab. (c) Stresses increased with decreasing value of the subgrade modulus. (d) Stresses produced by two and three-axle loads at the edges did not exceed those produced by single-axle loads although deflections were considerably larger: the maximum deflection under two-axle loads was twice that of the single axle. (e) For corner loads, two axles produced a maximum stress of only 90%, and three axles produced only 70% of the maximum stress of the single-axle load. (f) Stresses measured under more axles lay between those computed from the Arlington and Westergaard formulas; the Arlington formulas gave higher values. (g) Experimental results of the tests on the full-size slab agreed very well with those obtained on the model. (h) Stresses produced by two axles were a maximum when the distance between them was about five feet.

The reviewer considers the test results surprising because the measured stresses and deflections differed greatly from the computed ones. The reason for it evidently is the assumption of the theory of the elastic subgrade modulus ($p = ky$). Test results indicate clearly that maximum stresses and deflections are not proportional to the load, particularly not for low values of $k(k = 60)$. The theory of elastically supported beams and slabs calls for revision in the near future. J. Jáky, Hungary

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 47, 67, 71, 72, 76, 77, 84)

✎57. **Pierre Laurent, Jaques Valeur and Serge Sogroff, Foundations of mechanical strength of metals and alloys** (Les bases de la résistance méeanique des metaux et alliages), Paris: Dunod, 1947, 288 pp., 215 figs. Paper, 10 × 6.5 in. Appr. $5.

This book is intended to be a somewhat modern version of Schmid and Boas, *Kristallplastizität*, written for those interested in the extensive work on the plasticity of metals done in the present century. Unfortunately, as the authors admit in the introduction, the book covers most thoroughly recent work carried out east of the English Channel and makes no attempt to expound the developments carried out since 1935 in England and the United States. For example, although there is much discussion of theories of plastic flow in single crystals, the work of the group at Bristol in England, and of the reviewer and others in this country is omitted entirely. In spite of this shortcoming the book has considerable value since it provides one of the best recent reviews of continental work. The chapter headings follow:

Generalities: The metallic crystals, Elasticity, Thermal agitation in crystals.

Plasticity of single crystals: Geometrical study of the plastic deformations of single crystals, Experimental methods of studying slip and twinning, Practical study of the plasticity of single crystals, Influence of alloying elements, temperature and velocity of testing on the plasticity of single crystals, Recovery, recrystallization and asterism, Practical studies of twinning of single crystals, Rupture, Theories of the plasticity of crystals, A recent theory of plasticity of crystals, Effects accompanying plasticity of crystals.

Plastic deformation of industrial alloys: Geometrical study of the deformation of polycrystals, Experimental study of the deformation of polycrystals, Cohesion and resistance to glide in the polycrystal, Recovery by "redistribution" and by "sublimation," General characteristics of the resistance of metals, States of homogeneous tension, Study of nonhomogeneous plastic deformation, Mechanical tests: load, hardness, shock, Tests of flow, Related phenomena, Tests of fatigue, Study of textures.

On the whole, the authors make no attempt to develop the underlying picture of the process of plastic flow on the basis of modern atomic theory, but are content with semiphenomenological viewpoints. Frederick Seitz, USA

58. **Hisao Mii, On the torsion of materials with large longitudinal permanent set** (in Japanese), Trans. Soc. mech. Engrs. Japan, 13, no. 43, 15–21 (Jan. 1947).

Test pieces of duralumin, mild steel, and hard steel were stretched beyond the proportional limit, and thereafter subjected to torsion. The effect of the degree of longitudinal working on their behavior after yielding is investigated.

S. Moriguti, Japan

59. **A. Nádái, The flow of metals under various stress conditions,** Proc. Instn. mech. Engrs. 157, 121–160, 8 plates (1947).

In the first part of this lecture the author reviews the fundamental notions related to finite strains, introducing the strain variable $\lambda = (1 + \epsilon)^2$ which he calls "elongation" and with which he attempts an extension of the stress-strain relation from the range of infinitesimal to that of finite strains. Considerable attention is given in this part to the graphical representation of finite strains in the $\lambda\gamma$ plane (ϵ and γ being normal and shear strains), and also in a $\lambda'\gamma'$ plane where a change in variables allows the use of circles similar to Mohr's.

In a second part several examples are reviewed and the author makes clear the advantage of the analogy between plastic and elastic relationships, in particular in the case when the resultant strain tensor is coaxial with the stress tensor. He also very systematically reviews here the equilibrium conditions for plastic, viscous, firmoviscous and elastoplastic materials. The mathematical analogies are carefully pointed out, helping considerably in the understanding of a very complex field.

In the third part of the paper the author reports on some applications of mechanics of plastic materials to engineering problems, especially those connected with postwar efforts in the plastic forming of metals. The forces necessary for forging artillery shells

are here calculated in detail, and a simplified shell theory is developed in connection with the calculation.

In the last part of the paper some results of the following tests carried out by the author and by some of his associates for the Navy Department are summarized: flow and fracture of copper and of medium carbon steel; propagation of the yield zone along a bar; and increase of yield and flow stress with the rate of strain.

The figures are very clear and the photographs of the tests are excellent. More than 100 bibliographic references are given.

The paper will certainly prove to be extremely useful as a survey of plasticity studies. It would be easier to read if the same quantity were represented by the same symbol. A more systematic use of subscripts would help in this respect. "Octahedral" is represented, for instance, by the subscript "*oct*" as well as by "*o*," although other meanings have also been given to "*o*," and in the second part of the paper octahedral normal stresses and strains are symbolized without any subscripts.

A. J. Durelli, USA

60. P. M. Ogibalov and M. A. Loginova, On the dependence of the strains in a rapid deformation under impulsive loading beyond the elastic limit (in Russian), Vestnik Moskov. Univ. no. 5, 3, 39–58 (1948).

The analysis of a dynamic compression test is considered, in which a mass strikes a cylindrical specimen and compresses it axially under conditions of simple compressive stress. Incompressible deformation is considered with the specimen remaining cylindrical, effects of friction and inertia of the specimen being neglected. A strain-rate effect is considered giving the flow stress $\sigma = \sigma_0 \log \dot{\epsilon}/\dot{\epsilon}_0$ for $\dot{\epsilon} \geqq e\dot{\epsilon}_0$, $\sigma = \sigma_0$ for $\dot{\epsilon} \leqq e\dot{\epsilon}_0$, where $\dot{\epsilon}$ is the rate of increase of natural strain and σ the true stress. A differential equation is obtained for the variation of the length of the specimen with time after impact. Some simplification is obtained by using nominal strain based on the initial specimen length, and the analysis is developed to include the effects of work hardening. The solution for the final strain is obtained for the case of no work hardening. Numerical results show agreement with experiment, as also does a theory based on a linear, rather than logarithmic, strain-rate effect.

E. H. Lee, USA

61. Giulio Pizzetti, Contribution to the study of De Saint-Venant's problem in the elastoplastic (in Italian), Pont. Acad. Sci. Comment. 11, 1–30 (1947).

The author applies Colonnetti's principle [Pont. Acad. Sci. Comment. **2**, 131–150, 309–319, 439–514 (1938)] to the analysis of stresses and strains in elastoplastic bars. The following cases are treated in detail: pure flexure (rectangular cross section), torsion (circular cross section), bending by transverse forces (rectangular cross section), combined flexure and axial compression (rectangular cross section).

W. Prager, USA

62. J. E. Dorn, A. Goldberg and T. E. Tietz, The effect of thermal-mechanical history on the strain hardening of metals, Metals Technol. 15, T.P. no. 2445, 24 pp. (Sept. 1948).

Experimental work was done to determine the effect of thermomechanical history on the plastic properties of materials. The tests covered tension specimens, mainly of commercially pure aluminum, at 292, 260, 194 and 78 K, and disproved the existence of a relation $\sigma = \sigma(\epsilon, \dot{\epsilon}, T)$ (σ true stress, ϵ true strain, $\dot{\epsilon}$ true rate-of-strain, and T temperature), σ being found to depend upon the entire thermomechanical history; this result is to be expected, for the limitations of the above relation are well known.

The authors state their conclusions as follows: (1) The flow stress for plastic deformation of metals is not a simple function of the instantaneous values of the strain, strain rate and temperature; it is sensitive to the entire thermomechanical history. (2) At lower test temperatures higher work-hardened states are obtained for the same total strains; this factor is probably responsible for the well-known lowering of the recrystallization temperature as the metal is cold-worked at lower temperatures. (3) Simple history-modified strains did not yield equivalent work-hardened states. (4) The failure of the mechanical equation of state is perhaps partly due to the relaxation phenomena of low activation energy which can proceed at low temperature and high strain rates. (5) Future progress in determining the plastic properties of metals is largely dependent upon better understanding of the structural changes accompanying cold work and their effect on the flow stress.

H. G. Hopkins, England

Failure, Mechanics of Solid State

(See also Revs. 51, 59, 70, 83)

63. D. J. McAdam, G. W. Geil and F. J. Cromwell, Flow fracture, and ductility of metals, Trans. Amer. Inst. min. metall. Engrs. 175, 306–336 (1948).

The paper reports tension tests on notched specimens of copper, monel metal and ingot iron. Attention is focused mainly on the stress and strain when a fracture crack is initiated; the method used for determining this moment seems to have little justification. The fracture stress and strain are mean values across the current notch section. By comparing the results from notches of different depth and angle the authors discuss the influence of the state of combined stress on fracture. It appears to the reviewer that the detailed and involved deductions of the authors are misplaced in view of present ignorance about the nonuniform distribution of stress and strain in a notched specimen.

Rodney Hill, England

64. G. Vedeler, Brittle fracture in ship plate, Engineering 167, 289–290 (Apr. 1, 1949).

The author discusses the brittle fracture of mild steel plates due to triaxial stresses. Such fractures occur at places where some discontinuity such as a notch or a flaw in a weld is present, and may occur even at temperatures well above the transition temperature. On the basis of Maxwell's theory of shear energy and the strength theory of Mariotte and Poncelet, the author constructs a diagram defining the regions in which brittle fracture due to triaxial stresses may occur. This diagram shows that brittle fracture can occur only when all the principal stresses are tensile, as in the case of a considerable stress concentration due to a notch. By means of this diagram it is possible to determine the triaxiality ratio $T_r = (\sigma_1 + \sigma_2 + \sigma_3)/(3\sigma_1)$ which a notch must possess to make brittle fracture possible.

M. M. Gololobov, Czechoslovakia

65. V. D. Robertson, The effect of tensile and compressive stresses on the corrosion of an aluminum alloy, Trans. Amer. Inst. min. metall. Engrs. 175, 428–434 (1948).

24S aluminum alloy strips were solution heat-treated and aged, some at room temperature and some at 170 C to induct intercrystalline corrosion. Bending stresses were imposed by fixed deflections during exposure to a sodium-chloride and hydrogen-peroxide solution. Wax protected the specimens from attack on the compression or tension side as desired. Damage was measured by loss in tensile properties.

Compressive stress did not affect either the type or rate of attack when the metallurgical conditions made intergranular corrosion possible. This agrees in general with findings of Gurney and Borysowski (see Rev. **484,** Apr. 1949), and confirmed

a general belief. Other findings, as on the influence of tensile stresses on intergranular corrosion, confirmed previous reports.

A small inhibitory effect was found for compressive stresses on specimens not susceptible to intergranular corrosion. Repeated flexure tests may in some cases be far more sensitive than tensile tests in detecting and measuring light corrosive attack. Perhaps a more spectacular result could have been obtained by using such an evaluation test for the acceleration or inhibition of pitting corrosion of specimens aged at room temperature.

J. A. Kies, USA

66. **F. E. Richart, Jr. and N. M. Newmark, An hypothesis for the determination of cumulative damage in fatigue,** Proc. Amer. Soc. Test. Mat. 48, 767–800 (1948).

The hypothesis presented by the authors provides a method for predicting the endurance life of a machine part subjected to any arbitrary overstress cycle pattern. The presentation of the method assumes the existence of a damage-cycle relation during the process of forming a fatigue crack at any overstress, this relation being different at each stress level. No attempt is made to use reduction in endurance life to indicate the degree of damage sustained.

The process of failure at different stress levels was assumed to be representative by various curves on a damage-cycle ratio (D-R curves) diagram. These curves were used in computations for endurance life under various stress-cycle patterns, and from this it was observed that relative D-R curves were sufficient to give reasonably correct estimates of endurance. To make the method applicable to machine parts an experimental method of determining the relative D-R curves is presented. The analysis was verified by actual fatigue tests of specimens which were subjected to numerous cycles of different repeated stresses.

The present hypothesis is an extension of Miner's work in which it was proposed that the life of a specimen or part is governed by a relation of the cycle ratios at the various stress levels such that the sum of the cycle ratios equals unity. The accuracy of the present procedure for combining the cumulative damaging effects depends upon the experimental determination of the D-R curves, and cannot be used unless it is possible to run a number of fatigue tests to determine them. Actual application of the method to machine parts would be very laborious, and it will probably not be of much practical use unless some simpler approximation can be devised.

H. R. Neifert, USA

Design Factors, Meaning of Material Tests

(See also Revs. 9, 66, 75, 76, 80)

67. **Ya. N. Rabotnov, Evaluation of machine parts for creep** (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk no. 6, 789–800 (June 1948).

This discussion of creep in machine parts is based on theories of creep in a tensile specimen. The theory of constant creep [$\epsilon = S(\sigma)t$ where ϵ is elongation, σ stress and t time] is rejected as contrary to experimental evidence. The theory of hardening [$\Phi(p, \dot{p}, \sigma) = 0$, where p is plastic deformation $= \epsilon - \sigma/E$, and E is modulus of elasticity] is rejected as too complicated for mathematical calculations. The experimental evidence shows that the expression $\sigma = \varphi(\epsilon)\tau(t)$ (theory of aging) is a good basis for calculation with $\tau(t) = (1 + at^{\beta})^{-1}$ and with $\beta \backsimeq 0.3$. The agreement with experimental results is better than for the expression $\dot{p} = S(\sigma)\dot{T}(t)$ of Davenport.

As for nonlinear stresses, there is an almost complete lack of experimental evidence. Poisson's ratio is about $\frac{1}{2}$ at higher temperatures. It is assumed that: (1) there is no change of volume; (2) the deviators of stresses and of strains are proportional; (3) the intensities of stresses σ_i and of strains ϵ_i, as given by the formulas $\sigma_i^2 = (1/2)\{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\}$ $\epsilon_i^2 = (2/9)\{(\epsilon_x - \epsilon_y)^2 + (\epsilon_y - \epsilon_z)^2 + (\epsilon_z - \epsilon_x)^2 + (3/2)(\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{zx}^2)\}$, depend on each other in the same manner as the stress and the strain in a simple tensile test. For (2) it is found that

$$\sigma_x - \sigma = (2\sigma_i/3\epsilon_i)\epsilon_x, \ \tau_{xy} = (\sigma_i/3\epsilon_i)\epsilon_{xy}, \text{ etc.},$$

where $\sigma = (1/3)(\sigma_x + \sigma_y + \sigma_z)$. For (3), in the case of stationary creep, $\sigma_i = \varphi(\epsilon_i)$ and for the theory of aging $\sigma_i = \varphi(\epsilon_i)\tau(t)$.

Calculation in both cases is similar to calculations of hardening by plastic deformation. The author shows an application to the case of bending of a rectangular specimen and of thin and thick tubes. Graphical methods of evaluation must generally be used.

George Masing, Germany

Material Test Techniques

(See also Revs. 9, 51, 72, 75, 81, 179)

68. **Henry B. O. Davis, The TMB type-15K160 strain indicator,** David Taylor Model Basin Rep. no. 648, 29 pp. (Oct. 1948).

The article describes an electronic strain indicator for recording static or dynamic strains. Variable strains can be recorded at frequencies from zero to 2000 cps. Two 120-ohm Type-A SR-4 wire resistance gages are used and a sensitivity range of 0.25 to 10.0 milli-inches per inch is obtained. The instrument utilizes a 15-kc carrier voltage which is divided and modulated by the strain-gage bridge unbalance, and then recombined to provide the relatively high current necessary to drive the high-frequency galvanometers. A cable-length switch is provided to compensate for the resistance in the cable in the bridge circuit due to the attenuation at the high carrier frequency. A built-in calibration circuit provides a rapid and accurate check of the system. Installation, operation and circuits are discussed in detail, and circuit diagrams, parts lists, photographs, and a short bibliography are included.

William B. Stiles, USA

69. **W. S. Campbell, The TMB type 5A strain indicator,** David Taylor Model Basin Rep., no. 645, 42 pp. (Oct. 1948).

A carrier-type strain indicator using 2, 120-ohm Type-A SR-4 strain gages is described. The instrument provides a straight line response for frequencies from 0 to 1200 cps and has an error of only 5% at 1500 cps. The range of strain measurement is 0 to 2500 microinches per inch, and by a modification of the circuit this can be extended to 60,000 microinches per inch. The instrument features a built-in calibration circuit and a null detector, using a magic-eye tube, which provides for both resistive and reactive balance of the bridge circuit. The article describes a power unit which will operate two indicators simultaneously, and a special 5000-cps oscillator which can provide the carrier supply for six indicators simultaneously. Operating, checking and repair instructions are included as well as complete schematic diagrams and parts lists.

William B. Stiles, USA

70. **Toshio Nishihara and Toshiro Yamada, The fatigue strength of metallic materials under alternating stresses of varying amplitude** (in Japanese), Trans. Soc. mech. Engrs. Japan 14, 6–18 (Oct. 1948).

The authors have developed a theory of fatigue strength under alternating stresses with variable amplitude, and tested the theory by some experiments [Trans. Soc. mech. Engrs. Japan 10, no. 38, part 1, 23–27 (Feb. 1944)].

The theory gives

$$N_t = n_0/({}_0\int^{n_0} dn/N),$$

where N_t is the total number of alternations up to the failure, n_0 the number of alternations during one period of variation of amplitude, N the number of alternations up to the failure under any stress σ, n the cumulative number of alternations (that of stresses larger than σ) during one period.

The present paper reports further extension of the experiments. A rotary bending-test machine of the Ono type was used with three kinds of special loading equipment, the amplitude of alternating stresses being varied either sinusoidally or in such a way that the frequency curve approximated the actual one for loads upon aircraft. Flat as well as notched specimens were tested, made of mild steel, hard steel, and duralumin. The results of the experiments are in good agreement with the theory.

S. Moriguti, Japan

71. R. W. Asmussen and Frode Andersen, Studies on the elasticity of cellulose fibers, I (in English), Trans. Dan. Acad. tech. Sci. no. 5, 3–32; no. 6, 3–14 (1947).

In no. 5 an electronic stress-strain apparatus, modified from the design of Buchthal [F. Vidensk. Selsk. Biol. Medd. XVII, no. 2 (1942)] is employed to investigate single cotton fibers. A condenser with a moving plate is the stress-measuring element, and is connected with a high-frequency circuit whose output drives a d-c amplifier, which then records on a galvanometer (for static experiments) or an oscillograph. Dynamic and static Hooke's-law constants are obtained as functions of humidity, elongation and rate of elongation. Elastic and viscous effects are not clearly separated.

In no. 6 similar measurements were made on an unspecified rayon. While the length-tension diagrams of cotton were straight lines, those for rayon were of varying character with differing humidity. The simple Newtonian law is not adequate for the viscous force, and other models are discussed, but not applied.

George Halsey, USA

Mechanical Properties of Specific Materials

(See also Revs. 41, 57, 62, 63, 64, 71)

72. P. W. Bridgman, The linear compression of various single crystals to 30,000 kg/cm², Proc. Amer. Acad. Arts Sci. 76, 89–99 (Oct. 1948).

The linear and volume compression of single crystals of fifteen salts and one metal, magnesium, are reported, from tests by improved precision apparatus to pressures of 30,000 kg per cm². All crystals tested either had rotational symmetry, requiring the determination of two constants of linear compression, or were orthorhombic, requiring the determination of three constants. The author indicates that the reason for making the tests with the apparatus employed (the most sensitive and accurate now available) to such high pressures was to take advantage of the high accuracy to determine the changes of compressibility with pressure. No attempt is made to assess the physical significance of the data.

Most of the substances tested were conventional in behavior. However, two had markedly unusual features: $NiSO_4 6H_2O$, which had a very marked transition of the second kind in one direction; and benzil, which had a more complicated anomaly. Below 20,000 kg per cm² the length in the x-direction of $NiSO_4 6H_2O$ decreased nearly linearly with pressure at a rate about 40% of that in the z-direction. Near 20,000 kg per cm² there was a rapid increase in compressibility in a narrow pressure interval, the maximum compressibility being approximately 20 times the original. Both directions a and c were anomalous for benzil, and the anomalies were not connected as would be expected from thermodynamic considerations for a hydrostatic compression. In the c-direction an abrupt *increase* in length occurred at 22,500 kg per cm² followed later by an abrupt decrease at 28,500 kg per cm². At 12,500 kg per cm² there was a cusp-like increase in compressibility in the a-direction.

William M. Findley, USA

73. Hermann Schottky, The increase in tensile strength of austenitic manganese steel alloy (in German), Arch. Eisenhüttenw. 19, 55–64 (1948).

This paper discusses a relatively large number of tests on nickel and nickel-chrome steels with varying amounts of manganese. The paper shows that the tensile strength generally increases for increasing amounts of manganese up to approximately 14% without significant adverse effect upon the elongation at fracture of the material. Various plots of stress vs. reduction and area both before and at fracture are shown for various cold-work conditions of manganese bearing steels.

R. G. Sturm, USA

74. J. W. Freeman, E. E. Reynolds, D. N. Frey, and A. E. White, A study of the effects of heat treatment and hot-cold-work on properties of low-carbon N-155 alloy, Nat. adv. Comm. Aero. tech. Note no. 1867, 61 pp. (May 1949).

The properties of low carbon N-155 alloy as measured by tensile tests at room temperature and rupture tests at 1200 F are reported. The influence of solution treatments, aging treatments, and hot-cold-work were studied. Optimum treatments were determined based on the yield strength for a 0.02 per cent offset at room temperature and rupture properties at 1200 F.

Joseph Marin, USA

75. H. L. Burghoff and A. I. Blank, Fatigue properties of some coppers and copper alloys in strip form, Proc. Amer. Soc. Test. Mat. 48, 709–736 (1948).

There exist only a few published results of fatigue tests of nonferrous metals in thin sheet or strip form for flat springs. The present paper presents results of an extensive research program, concerning the fatigue properties of three types of copper, five copper-zinc alloys and four other copper-base alloys. Especially the influence of the reduction of area by rolling (21, 37 and 60%) and of the content of P, Pb, Sn and Zn on the fatigue strength were studied. Moreover the variation of fatigue strength with grain size (0.020 to 0.120 mm) and specimen position (specimens cut with length parallel, 45 and 90 deg to rolling direction) were determined for several materials. The test materials were rolled strips with a thickness of 0.032 in., from which the samples were taken without an additional finishing of the surface. In this investigation two multispecimen machines (40 samples, n = 900 cpm) were used and the Wohler tests were made under reversed bending load up to a final number of 100 million cycles.

The results of the fatigue tests are plotted for the different test groups and materials in 75 stress-cycle curves. The values of the fatigue strength at 100 million cycles are taken from these curves and the dependence of these values on the studied influences (tensile strength, reduction of area by rolling, content of P and Zn, angle between specimen axis and rolling direction, grain size, etc.) are discussed in a detailed manner.

M. Hempel, Germany

76. G. R. Gohn and W. C. Ellis, The effect of small percentages of silver and copper on the creep characteristics of extruded lead, Proc. Amer. Soc. Test. Mat. 48, 801–840 (1948).

The authors have reported the results of their very thorough investigation of the creep characteristics of six compositions of lead sheathing cable. The tests were made on 0.500-in. wide speci-

mens which were cross-milled from flattened sections of 0.140-in. thick pipe. The alloys investigated varied in silver content from 0 to 0.018%, and in copper content from 0 to 0.061%, and were approximately 18 months old at the start of the tests.

The data presented indicate that the creep rate of lead is markedly decreased at stresses of 400 psi and above for additions of silver up to 0.010%, but that the addition of more than 0.010% of silver had only a small effect toward improving the creep resistance at any stress level. For additions of silver up to 0.010%, the improvement in creep resistance at 400 psi was only about $\frac{1}{4}$ that observed at higher stresses. At stresses of 250 psi and below the data indicate that the silver has only a small effect on the creep rate. No appreciable improvement in creep rate was obtained from the addition of 0.061% of copper to the higher silver alloy.

The importance of obtaining creep data on lead cable sheathing alloys is clearly indicated by the amount of controversial data published by the various investigators during the past twenty years. The controversial nature of available data is due in part to the effect upon the chemical and physical properties of the resulting alloys of small additions normally considered to be well within the impurity range. In the stress range of 100 to 300 psi—the range of engineering interest—very little information is available on account of the long period of time the tests must be continued before reliable creep rates are established. The need for more data at the lower stress levels is indicated by the authors in concluding their paper: "Whenever possible, creep rates should be obtained for the stresses of interest from test data rather than by extrapolation from data taken at higher stress levels."

H. R. Neifert, USA

77. R. F. **Hanstock, The effect of vibration on a precipitation-hardening aluminum alloy,** J. Inst. Metals 74, 469–492 (May 1948).

The dynamic properties of the precipitation-hardening aluminum alloy hiduminium RR56 were investigated by determining the relation between the specific damping capacity and strain amplitude for various metallurgical conditions of the alloy. The results suggested to the authors that strain-dependent specific damping capacity might result from dislocations generated under the combined influence of strain and thermal fluctuations. The factors which the author considered to be mainly responsible for the dynamic behavior of the alloy were: an internal field of strain, probably originating from foreign atoms in solution or dispersed in the aluminum lattice, and a strain-concentration factor depending on the degree of precipitation.

To support this idea the author derived an equation for specific damping capacity from the Boltzmann-Maxwell equation for distribution of energy and the assumption that the energy dissipated per cycle is due to translation of dislocations.

The resulting equation for the specific damping, $\Delta E/E$, is: $\Delta E/E = B \exp [-VGq^2 (\phi_m - \phi)^2/2LkT]$ where ϕ_m is the external shear strain corresponding to thermal fluctuation sufficient to raise the applied shear strain ϕ to the value necessary to generate a dislocation, q is a strain-concentration factor, T is the absolute temperature and the other values are constants described in the paper.

It seems to the reviewer that some of the concepts and assumptions used in this derivation may not be entirely justifiable; so that the derivation is not entirely satisfying, although the agreement with the data presented is quite convincing.

The author reports that vibration at high-strain amplitudes accelerates precipitation, and if sufficiently prolonged produces changes in metallurgical condition normally associated with temperature aging and annealing. It is suggested that because of its directional nature, vibration is probably selective in its action on the grains of a polycrystalline metal and consequently does not affect the average static mechanical properties to a significant extent. However, the grains affected by vibration determine the fatigue properties, and it is suggested that the endurance under a given strain amplitude may be inferred from the change in damping capacity during vibration. William N. Findley, USA

78. **S. I. Liu, J. J. Lynch, E. J. Ripling and G. Sachs, Low cycle fatigue of the aluminum alloy 24ST in direct stress,** Trans. Amer. Inst. min. metall. Engrs. 175, 469–496 (1948).

A special design of a tensile specimen was subjected to reversed strains of such magnitudes that failure occurred in 7 cycles or less. Duplicate specimens were also carried through only limited portions of their normal fatigue life, at which time stress-strain curves were run to fracture. This permitted analysis of the progressive changes in flow and fracture characteristics with cycling. These changes are plotted in a number of ways; for example, true stress and strain, and fracture stress or fracture strain (retained ductility) as the ordinate, and number of cycles or test strain level as the abscissa.

Fracture stress decreased with both the magnitude of strain and the number of cycles, reaching for the maximum number of cycles in a particular test series a value slightly below the applied tension stress. Fatigue failure finally took place as a result of progressive deterioration of the metal, in spite of strain hardening in the early cycles.

A correlation with conventional fatigue data (at millions of cycles) is discussed, and will be studied further after the compilation of additional data in the range of a few thousand cycles. A discussion is included. Charles W. Gadd, USA

79. **B. M. Loring, W. H. Baer and G. M. Carlton, The use of the Jominy test in studying commercial age-hardening aluminum alloys,** Trans. Amer. Inst. min. metall. Engrs. 175, 401–413 (1948).

An experimental method is presented for the study of effects of cooling rate on age-hardening phenomena of aluminum alloys. As the rather heavy L-type Jominy specimen is used with the experiments, the method is important for alloys to be used in heavy sections. The following alloys were investigated: 148, 248, 628, 758 and R-301. The specimens were given solution treatment in a furnace as recommended. Thereafter they were end-quenched in water. The hardness of the specimens was determined as function of the distance from the quenched end. Furthermore the cooling rate during quenching was measured by the aid of a number of thermocouples on one surface of the specimen and a Hathaway S-12 oscillograph. There seems to be a critical rate of quenching. High rates of cooling cause progressive decrease in hardness of the quenched alloys. Low rates of cooling produce relatively small changes in hardness. With one exception the widely different rates of cooling had little effect on the hardness attained in aging the quenched specimens.

R. G. Boiten, Holland

80. **Robert F. Blanks and Douglas McHenry, Plastic flow of concrete relieves high-load stress concentrations,** Civ. Engng. 19, 40–43 (May 1949).

The authors claim that discontinuities have surprisingly little effect on the ultimate strength of concrete structures, and they show results of compression tests on solid blocks and on blocks with holes whose loading capacity at 28 days' age is essentially the same. The authors explain the phenomenon by the readjustment of the stresses produced by the plastic behavior of concrete.

When the concrete is one year old, however, the tests show a

pronounced difference in the loading capacity of the solid and hollow blocks. This fact seems to prove that stress concentration should be taken into consideration. The tests indicate also that discontinuities have a definite influence on the cracking of concrete. In structures where cracks cannot be allowed, the tensile strength of concrete may be more significant than the compressive loading capacity. A. J. Durelli, USA

81. M. B. Le Camus, Experimental research on the deformation of concrete and reinforced concrete (in French), Bâtim. Trav. publics no. 32, 1–18; no. 33, 1–19; no. 34, 1–15 (Jan. 1947).

The paper deals with investigations carried out at the testing laboratories of the Institut Technique du Bâtiment et des Travaux Publics in Paris. The elongations are measured by means of the Coyne's *témoin sonore* (electroacoustical method). The measuring technique is described and the corrections, due to temperature, bending of the pivots supporting the vibrating cord, and to creep of the latter, are discussed. Some differences were found between the electroacoustical and the mechanical measurements of the elongations. Only one kind of concrete was used in the tests.

The paper is in three sections. In the first section the deformation of concrete under nonpermanent loads is investigated (stress-strain diagram, elastic modulus for compression and tension and its variation with the loading range and the concrete age, Poisson's ratio, modulus of torsion). In the second section the deformation of concrete under permanent loads is studied: elastic and plastic deformation under impact load, creep and shrinking of concrete and their stabilization with time, residual deformation at the removal of the load, and its reduction with the time until stabilization. Influence of aging and seasoning conditions are investigated. The third section describes tests with reinforced-concrete specimens, which have the purpose of investigating the deformations of reinforced concrete under nonpermanent and permanent loads, and the influence of the reinforcing bars on the shrinkage.

According to the author's final remark no decisive conclusions can be obtained from these researches; they have, however, suggested the broad lines of the phenomena and laid the basis for further systematic investigations. Giulio Ceradini, Italy

82. E. W. Scripture, Jr., F. B. Hornibrook and D. E. Bryant, Influence of size grading of sand on air entrainment, J. Amer. Concr. Inst. 20, 217–228 (Nov. 1948).

Mixes of sand and water alone, 1:4 and 1:2 mortars, and concrete mixes were made with and without air-entraining agents and their air contents determined. While size grading of the sand had a very large influence on air entrainment in a mixture of sand and water alone, this effect was smaller in mortars and very small in concrete mixes. M. Reiner, Israel

83. Tetsugoro Futagami, On the mechanism of disruption of glass (in English), Kyushu Univ. Res. Inst. Elast. Engng. Rep. no. 2, vol. 4, 17–25 (1947).

The modes of fracturing of glass rods under tension and torsion, as well as of glass plates under a concentrated load are described. In each case three different parts of the cracked surface were found: glossy, matt, and multiple cracked. The presence of the different types of surfaces was attributed to different distributions of nuclei of the cracks, which were in turn influenced by the strain rate.

The strength of soda glass was reported for tension, compression and torsion. By use of Mohr's circle the direction of a cracked surface was predicted and found to be a plain surface, a logarithmic cylindrical surface, or a surface of revolution of a logarithmic spiral, for tension, torsion and a concentrated load respectively on a flat plate. These results agreed with the author's observations.

There are no references and the meaning is not always clear. William N. Findley, USA

Mechanics of Forming and Cutting

(See also Revs. 59, 75)

84. Werner Lueg and Josef Haas, The calculation of the required power in drawing with counter pull (in German), Arch. Eisenhüttenw. 19, 137–143 (1948).

The authors draw heavily upon the analytical work by F. Korber and A. Eichinger who in turn refer back to W. Lueg and A. Pomp and present a considerable amount of data comparing experimental results with the analytical results formerly published. They show optimum die angles for minimum drawing tension for different per-cent reductions in the drawing of steel wire. The authors obtain very consistent results and conclude that practical results can be obtained from the Korber-Eichinger equations for reductions in cross section ranging from 5 to 50%, provided the optimum die angles are used. R. G. Sturm, USA

Hydraulics; Cavitation; Transport

(See also Revs. 172, 180, 183)

85. J. Lamoen, Tests on reduced models of weirs, I: Influence of scales (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale III, 359–503 (1948).

The author's publication deals in great detail with a series of tests carried out on the spillway of Grosses Battes (on the River Ourthe, Belgium) and on the spillways of the Eupen Dam (Belgium). The whole series of measurements is published and critically reviewed. The Grosses Battes spillway has been tested on several models at different scales (1:50, 3:50 and 1:100), and on a second series of models at scales 1:100, 1:66.7, 1:50, 1:30 and 3:50. These models were of smoothed concrete, smoothed stone, or bronze. Similar model tests were carried out for the Eupen Dam spillway.

The main question was the law of extrapolation of the test results (scale effect). A first series of tests was in agreement with the laws described by Escande, the larger models giving greater discharges than the smaller ones when reduced to the same units by application of Froude's law. A law was found permitting final extrapolation. Another series of tests led to different conclusions.

Unstable series of observations (variations of about 5%) were found, thus confirming some previous examinations of O. Kirschmer (Mitt. Hydr. Inst. Muenchen, 1928).

Charles Jaeger, England

Incompressible Flow: Laminar; Viscous

(See also Revs. 35, 95, 120, 130, 131, 132, 133, 160, 170, 180, 183)

86. H. Julian Allen, General theory of airfoil sections having arbitrary shape or pressure distribution, Nat. adv. Comm. Aero. Rep. no. 833, 23 pp. (1945, publ. in 1947).

This presents a two-dimensional theory of thin airfoils of small camber, which may be used to calculate either the velocity distribution corresponding to a given airfoil shape or vice versa.

The latter problem is of current importance in the development of laminar-flow airfoils. The procedure employed is to combine the solution for a cambered airfoil of zero thickness at an angle of attack with that for an airfoil of zero camber and angle of attack but with the proper thickness distribution (called the base profile).

The first of these solutions is obtained from Glauert's thin-airfoil theory, involving replacing the mean camber line by a distribution of vortices along the chord. The solution for the flow around the base profile is obtained analogously by replacing the airfoil by a distribution of sources and sinks along the chord. The integral equation for each problem is solved by expanding the unknown vortex and source distributions in Fourier series, following Glauert. These are found to be summable, leading to closed solutions for the unknowns in terms of integrals of the knowns. The resulting integrals for the camber-line problem are essentially those of Munk's thin-airfoil theory, which can be obtained directly.

The author suggests that the solution of the camber-line problem may be corrected for airfoil thickness by multiplying the pressure at any point by the ratio of the local velocity to the free stream velocity at that point, as obtained in the solution to the base profile. Since this correction is made to the final pressure, and not used in the determination of the vorticity distribution, it must be considered semiempirical, but it brings the calculated results into very good agreement with a more exact theory (conformal transformation of the airfoil into a circle) and with experiment.

In the solution of the base-profile problem, it is found that the method employed breaks down at the leading edge of an airfoil, because the slope of the profile becomes infinite there. To circumvent this difficulty a "reference base profile" is chosen which has the same characteristics as the given profile at the leading and trailing edges (for which the pressure distribution is easily determinable), and changes in pressure distribution are calculated from that of the "reference" profile. Since the base-profile theory is linear, superposition is possible.

A. H. Flax, USA

87. **Gunther Kempf, Moments of cambered round bodies,** Nat. adv. Comm. Aero tech. Memo. no. 1227, 15 pp. (Aug. 1949).

Experimental results are presented for the pitching moments and position of the centers of pressure of a series of cambered round bodies derived from a torpedo-like body of revolution. The data were obtained by towing the bodies fully immersed in water at speeds of 1 to 2.5 meters per second, corresponding to Reynolds numbers of 6.7×10^5 to 16.65×10^5. Data were also obtained showing the effect of fins at the rear of the body of revolution upon the longitudinal stability of the body. Ed.

88. **A. D. Young and Miss E. Young, A family of streamline bodies of revolution suitable for high-speed or low-drag requirements,** Rep. Memo. aero. Res. Counc. Lond. no. 2204, 15 pp. (Aug. 1945, issued in 1948).

In a previous paper by Young and Owen [Rep. Memo. aero. Res. Counc. Lond. no. 2071 (July 1943)] a method was described of determining a streamline body of revolution yielding a velocity distribution approximative in form to a specified type. A simple type of velocity distribution considered there in detail ("roof-top") is chosen as basic type for the objective of the present paper. If, as the authors assume, the tail of the body should be pointed, the shapes derived are determined by three parameters.

The authors describe a process for the derivation of a particular family of bodies recommended where low-drag or high-speed requirements are paramount, and in all cases where smooth velocity distributions free from excessive localized peaks are desired. This family is based on a number of standard shapes with a fineness ratio of 0.2. As noticed by the authors, after calculating a number of bodies it is possible to derive with sufficient accuracy streamline bodies having other fineness ratios, by simply multiplying by a scaling factor. A table is given for the ordinates of the basic shapes for parameter ranges believed sufficiently wide for any particular purpose. In order to illustrate the ranges of pitch over which low-drag properties may be expected, examples are given of velocity distributions over the basic shapes at various angles of pitch. H. Görtler, Germany

89. **H. Parkus, On the calculation of air forces on an oscillating lifting surface** (in German), Anz. Akad. Wiss. Wien 85, no. 10, 91–102 (1948).

Transforming without hesitation surface integrals into integrals over the space occupied by a rigid body immersed in a fluid, the author arrives at formulas for the hydrodynamic force and moment on this body, which, applied to thin aerofoils, differ from the usual result by a useless (and well-known) second-order term.

J. H. Greidanus, Holland

90. **Garrett Birkhoff, Remarks on streamlines of discontinuity** (in Spanish), Revista Ci., Lima 50, 105–116 (1948).

When fast-moving liquid flows past a fixed obstacle (or when the obstacle travels through the fluid) a cavity filled with air or vapor is found to form behind the obstacle, and often a reentrant jet is observed at the downstream part of the cavity. Let p_f be the pressure in the free stream and p_c the pressure in the cavity. Then Prandtl's cavitation number is $K = (p_f - p_c)/\frac{1}{2}\rho V^2$. The author proves that for an axially symmetric reentrant jet (whether in two or three dimensions) the drag coefficient is $2\{1 + K + (1 + K)^{1/2}\}\sigma$, where σ is the ratio of the jet cross section to the obstacle cross section.

Considering further the penetration of fluid by a fluid jet when a slipstream interface is formed, it is proved that the speeds on either side of the interface are related by $v^2 = Av'^2 + C$, where A and C are constants. This relation holds not only for incompressible fluids but also for compressible fluids with similar polytropic equations of state, provided the exponent is the same for both fluids. L. M. Milne-Thomson, England

91. **R. Roscoe, The flow of viscous fluids around plane obstacles,** Phil. Mag. 40, 338–351 (Mar. 1949).

The author gives a solution of the equations of Stokes for the slow motion of a viscous incompressible fluid; ϕ being a harmonic function, this solution is expressed in the following way:

$$U = \phi - x\frac{\partial\phi}{\partial x}, v = -x\frac{\partial\phi}{\partial y}, w = -x\frac{\partial\phi}{\partial z}, p = -2\mu\frac{\partial\phi}{\partial x}. \tag{S}$$

Let ϕ be taken as the electric potential around a grounded conducting plane lamina placed in the plane Oyz. The Equation (S) represents then the flow of a viscous fluid around a thin plate of the same form as the conducting lamina. This idea is applied to the following problems:

(1) Flow of a viscous fluid around a thin plate placed perpendicularly to a uniform stream of velocity U. Let C be the electric capacity of a conducting lamina of the same shape as the plate; then the total normal force on the plate is $8\pi\mu\, CU$. All is therefore reduced to the determination of C, which is done by the author in certain particular cases. (2) Flow through an elliptic aperture in a thin wall; the rate of flow of fluid through the aperture is shown to be $2S^2/3\pi\mu s$ times the pressure difference between

two sides, S being area of the aperture and s its perimeter. (3) Plate rotating about an axis in its plane in a stationary fluid of infinite extent. In particular, the torque on an elliptic plate rotating about its minor axis is calculated. This result is applied to the study of a paddle viscometer described by Geddes and Dawson [Ind. Engng. Chem. 34, 163 (1942)].

Ratip Berker, Turkey

92. P. Y. Chou, The laminar mixing motion of two incompressible gases, Chinese J. Phys. 7, 96–101 (Dec. 1947).

The author investigates the differential equations for the motion of a mixture of two gases with the restrictions that thermal diffusion can be neglected and that the sum of numbers of molecules of both kind per unit volume is constant. The second restriction is called by the author the condition of incompressibility. Under these restrictions, the temperature does not enter into the calculation and the unknowns of the problem are the three components of the mean mass velocity of the mixture, the pressure and the number concentration of the components of the mixture. These five unknowns are determined by five equations, three dynamic equations, the continuity equation and the diffusion equation. [The author's statement that there are six unknowns and the need for the adiabatic equation of state for gases are difficult to understand.] The author then shows that, for mixing in plane jets, the prevailing similarity law is the same as that for one gas.

H. S. Tsien, USA

Compressible Flow, Gas Dynamics

(See also Revs. 136, 138, 144, 146, 147, 151, 152, 154, 158, 173, 174, 182)

93. C. C. Lin, An introduction to the dynamics of compressible fluids, Hdqtrs. Air. Mat. Comm. Dayton Tech. Rep. no. F-TR-1166-ND, 1–159 (Oct. 1947).

The monograph presents a survey of the classical background in gas dynamics and theory of compressible fluids, with emphasis on the mathematical development of the general equations and classical methods of solution. The work is notable for the mathematical completeness and generality in the discussion of some of the methods of solution, with inclusion of some of the new developments in technique.

Chapters I and II present the basic equations, with little discussion, and develop the one-dimensional solutions for propagation of plane waves, both infinitesimal and of finite amplitude. The discontinuous or shock wave is then discussed. In chapter III the one-dimensional equations are applied to a channel of slowly changing cross section, and a physical discussion of the flow in a nozzle with various exit conditions is given briefly.

The general three-dimensional equations are taken up in chapters IV and V. The Rayleigh-Jansen expansion for subsonic flows is treated, and a very complete and general development of the Prandtl-Glauert equations, applicable to both two- and three-dimensional problems, is given for the case of subsonic flows. The application of variational methods of solution is described, and a particular approximate application is presented. The hodograph transformations, including the Chaplygin equations are given. The small perturbation method is developed for supersonic flows in two dimensions or with axial symmetry, but not in the general three-dimensional case. The particular exact solution for flow around a corner is used to introduce the method of characteristics in two dimensions. No description of the three-dimensional problem is given. General solutions in terms of hypergeometric functions, particularly for mixed flows, are presented, including the Ringleb solution.

Chapters VI and VII present a discussion of flows with heat transfer and viscosity. The general equations are developed, and the one-dimensional form applicable to calculation of flow through a shock wave is shown. The boundary-layer approximations are given, and the method of solution for the case that the Prandtl number equals 1 is described. The relation between the heat transfer and shear force calculation is clearly presented, and a brief résumé of the Kármán-Tsien method of successive approximations is given.

Allen E. Puckett, USA

94. W. Weibull, Waves in compressible media. I. Basic equations. II. Plane continuous waves, Trans. roy. Inst. Technol. Stockholm no. 18, 38 pp. (1948).

This is an elegant exposition of elementary theory of ideal, one- or two-dimensional compressible flow (called "wave" by the author), in terms of the familiar Busemann variables

$$a \pm v,\ a = \int dp/\rho u$$

(u speed of sound, v particle velocity), and the differential operators

$$D_\pm = \partial/\partial t + (v \pm u)\,\partial/\partial x.$$

Unified formulas for continuous flow and shock waves result. Simple ("progressive") waves are discussed in more detail, including some basic interaction problems.

The dependence of a on the pressure is analyzed for a number of equations of state. The usual tables for shock wave computation are given.

A. D. Kafadar, USA

95. A. S. Predvoditelev, On a molecular-kinetic derivation of the equations of hydrodynamics (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk no. 4, 545–560 (Apr. 1948).

The author's objective is to deduce hydrodynamic equations from the kinetic theory of gases. He claims that Maxwell's hypothesis of mutual forces varying as the (-5)th power of the distance is sufficiently accurate to yield the correct form of all equations, and the first part of the paper is a recapitulation of Maxwellian arguments. He rejects Maxwell's assumption that the speeds imparted to the two participants in a collision are equal, and replaces it by assumptions of mixed molecular and phenomenological nature which the reviewer cannot understand. Thence he deduces equations of transfer, and adds new assumptions, including that of isentropic motion. (Reviewer's note: For viscous fluids, this assumption is generally inconsistent with the energy equation.) In the hydrodynamic equations he finally shows that the second coefficient of viscosity λ is related to the first μ by Busemann's equation $\lambda = (1-\gamma)\mu$, where γ is the ratio of specific heats, and the acceleration a^i is replaced by $(1-\beta)a^i + \beta[\partial v^i/\partial t - v^i \operatorname{div} v]$, where v^i is the velocity vector and β is a new coefficient which vanishes under Maxwell's assumption.

In the derivation there is no attempt to use mathematically acceptable methods, and no references to the extensive literature are included. As a first application, the author obtains the Bernoulli equation for steady irrotational flow when viscosity is neglected; according to his result, if $\beta < 1$ the density decreases as the speed increases, and there is an upper limit to the speed, while if $\beta > 1$ speed and density increase together, and there is no limit speed. For a second application, he observes that if $\beta > 1$, the potential equation, which he derives for plane isentropic gas flows in analogy to Chaplygin's, has only one instead of two real singularities in the hodograph plane.

C. A. Truesdell, USA

96. Jacques **Hadamard, Lectures on wave propagation and the equations of hydrodynamics** (in French), New York, Chelsea Publishing Co., 1949, 372 pp. Cloth, 9.5 × 6.3 in., 25 figs. $2.75.

This is a reprint of the pioneer and classic work on wave propagation and shock theory first published in 1903. Ed.

97. Stefan **Bergman, An initial value problem for a class of equations of mixed type,** Bull. Amer. math. Soc. 55, 165–174 (Feb. 1949).

The author considers a family of linear differential equations of mixed type (two variables), $\psi_{xx} \pm (\mp x)^n\psi_{yy} = 0$ for $x < 0$ and $x > 0$, respectively, which contain the Tricomi equation as a particular case. The function and its normal derivative are specified on the line where the differential equation is parabolic. The solutions are found in the form of definite multiple integrals. The domain of definition of the solution is discussed and for certain restricted boundary conditions the integral representations are simplified considerably. George F. Carrier, USA

98. M. A. **Biot, Loads on a supersonic wing striking a sharp-edged gust,** J. aero Sci. 16, 296–300 and 310 (May 1949).

Time histories of the pressure distributions, total lift, and mid-chord pitching moment on a restrained, two-dimensional airfoil entering a sharp-edged gust are presented as a function of Mach number. These results have been given in the Proc. roy. Soc. Lond., Dec. 1948, vol. 195, by W. J. Strang, and also in Nat. adv. Comm. Aero. tech. Note, no. 1621. Harvard Lomax, USA

99. I. I. **Eterman, Pressure distribution on bodies of revolution in high subsonic gas flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1222-IA (GDAM A9-T-41), 17 pp. (May 1949): see Rev. **344,** Feb. 1948.

100. S. V. **Falkovich, Plane motion of a gas at hypersonic velocity,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1221-IA (GDAM A9-T-40), 11 p. (May 1949): see Rev. **1247,** Aug. 1948.

101. M. M. **Munk, On supersonic flow patterns,** J. appl. Phys. 20, 302–305 (Apr. 1949).

As the author states, for a definite Mach number, given by $M^2 = 4/(3 - \gamma)$, all Mach lines in a steady plane potential flow of a perfect gas with expansion coefficient γ have zero curvature. The use and the significance of this relation are discussed, and the hypothetical expansion law consistent with straight Mach lines is deduced. Harvey Cohn, USA

102. B. M. **Kiselev, Calculation of one-dimensional gas flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1209-IA (GDAM A9-T-27), 30 pp. (Jan. 1949): see Rev. **681,** Apr. 1948.

103. P. **Germain, Theory of homogeneous motions and its application to certain delta wings in supersonic flow** (in French), Rech. aéro. Paris no. 7, 3–16 (Jan.-Feb. 1949).

The author gives a short account and continuation of a previously written voluminous paper which is said to appear soon [La théorie générale des mouvements coniques et ses applications à l'aérodynamique supersonique, Off. nat. Étud. Rech. aéro. Rep., Paris]. The author defines the homogeneous flows as ones for which the velocity potential is a homogeneous function of order n of the three variables x_1, x_2, x_3 (x_1 is the direction of flight), the conical flow being a homogeneous flow of order one. He uses these functions to analyze the supersonic flow around delta wings with sections given by the equation

$$x_3 = x_1(1 - x_1)\,[\lambda_1 x_1 + \lambda_2 x_1^2 + \ldots \lambda_{n-2} x_1^{n-2}]\,f(x_2/x_1).$$

In the first chapters the author assumes symmetry of the wing relative to the x_1x_3-plane. Later he drops this restriction in order to handle, e.g., the rectangular half-infinite wing, etc. At the end of the paper it is shown how electrical analogies may help to solve cases which would involve too much computation work.

I. Flügge-Lotz, USA

104. L. A. **Galin, A wing of rectangular plan form in supersonic flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1217-IA (GDAM A9-T-36), 20 pp. (May 1949): see Rev. **1152,** July 1948.

105. M. I. **Gurevich, On the problem of a thin triangular wing moving with supersonic velocity,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1216-IA (GDAM A9-T-35), (May 1949): see Rev. **1382,** Sept. 1948.

106. A. A. **Sokolov, On the solution of the nonlinear hydromechanical equation by the perturbation method** (in Russian with English summary), Vestnik Moskov. Univ. 1947, no. 4, 77–82 (1947).

The author treats the equation describing the (adiabatic) motion of a gas in a tube with one fixed end and with a freely moving piston of mass M. The differential equation reads: $\xi_{tt} = c^2(\xi_x)^{-\gamma-1}\xi_{xx}$. The initial conditions are: $\xi_t = 0$, $\xi_x =$ constant for $t = 0$, $0 \leqq x \leqq a$. The boundary conditions are: $\xi = 0$ for $x = 0$, $\xi_{xx} = (\mu/a)\xi_x$ for $x = a$. Here ξ is the amplitude x and t Lagrangian variables, c the acoustic speed for the (constant) initial state, γ the ratio of specific heats, a the initial distance from the piston to the fixed end, $\mu = m/(\gamma M)$, m being the mass of the gas [cf. A. E. F. Love and F. B. Pidduck, Philos. Trans. Roy. Soc. London. Ser. A. 222, 168–226 (1922)]. Assuming an expansion in powers of μ and neglecting μ^2 and terms of higher order, the author obtains an approximate solution of the form $\xi = \xi_1(t)\xi_2(x) + u$. Here $\xi_2 = x - (\mu/6)(x - x^3/a^2) + \ldots$; ξ_2 is determined by the relation

$$\int_1^{\xi} \xi_1^{(\gamma-1)/2}[\xi_1^{\gamma-1} - 1]^{-1/2}d\xi_1 = (2vk)^{1/2}t,$$

where $v = 1/(\gamma - 1)$, $k = c^2a^{-2}\mu[1 - \frac{2}{3}(1 + v)\mu/(2v)]$. For small values of t, $\xi_1 \approx 1 + kt^2/2$; for large values of t, $\xi_1 \approx (2vk)^{1/2}t$. The formulas for the additional term u are too complicated to be reproduced here. This term represents a standing wave which disappears as $t \to \infty$. L. Bers, USA

107. S. A. **Khristianovich, Approximate integration of the equations of supersonic gas flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1220-IA (GDAM A9-T-39), 18 pp. (May 1949): see Rev. **1383,** Sept. 1948.

108. F. **Reutter, Plane potential flow in the neighborhood of the speed of sound** (in German), Ingen.-Arch. 16, 299–306 (1948).

The author assumes a nearly uniform flow and a relation between the velocity and density which leads to the potential equation (1) $-\phi_x\phi_{xx} + \phi_{yy} = 0$ which is of mixed type. A Legendre transformation yields the linear equation (2) $-U\varphi_{vv} + \varphi_{uu} = 0$. Particular solutions of (2), especially those expressible by Bessel functions, are discussed and various possibilities of applying these to the computation of flows are described. No definitive

procedure is given, however, and the author promises numerical results at a later date. Equation (1) was used by von Kármán [(J. Aeronaut. Sci. 14, 373–402 (1947), p. 396] to derive a similarity law for transonic flows. The author seems to be unaware of the existing literature on the Darboux-Tricomi equation (2) and its applications to gas dynamics.

Courtesy of Mathematical Reviews L. Bers, USA

109. R. Siestrunck and J. Fabri, On the general equation of helical potentials in an ideal compressible fluid (in French), C. R. Acad. Sci. Paris 226, 1430–1431 (May 3, 1948).

The authors give the following general equation (in cylindrical coordinates r, θ, z) for the velocity potential φ of nonsteady flow of a perfect compressible gas:

$$a^2\Delta\varphi = \varphi'^2_r\varphi''_{rr} + (\varphi_\theta'/r^2)\varphi''_{\theta\theta} + \varphi'^2_z\varphi''_{zz} + \varphi''_{tt} + 2(\varphi'_r\varphi'_\theta\varphi^2_{r\theta}/r^2 + \varphi'_\theta\varphi'_z\varphi''_{\theta z})/r^2 + \varphi'_r\varphi'_z\varphi''_{rz}) - \varphi'^2_\theta\varphi'_r/r^3 + 2(\varphi'_r\varphi''_{rt} + \varphi'_\theta\varphi''_{\theta t}/r^2 + \varphi_z'\,\varphi'_{zt}),$$

where a denotes the velocity of sound. They remark that, to their knowledge, this equation had not been pointed out previously.

Among the fluid motions governed by this equation, the authors consider in some detail flows with helical potential surfaces. The equation is linearized for nearly uniform flows, and can then be solved by elementary functions. It is pointed out that in a subsonic flow the linearized potential of the perturbed flow does not depend upon θ and z independently, but upon the combination $\zeta = (\theta - z\omega/v)$, which had been used by Goldstein. Thus one obtains an equation analogous to the Goldstein equation with some compressibility effects included. It is thus found that compressibility does not affect the nature of the potential field but only the boundary conditions. Paul Lieber, USA

110. L. Howarth, The propagation of steady disturbances in a supersonic stream bounded on one side by a parallel subsonic stream, Proc. Camb. phil. Soc. 44, 380–390 (July 1948).

This paper deals with an analytical study of the propagation of a steady disturbance through a fluid consisting of two parallel semi-infinite streams, one supersonic and the other subsonic. The disturbance is considered to originate in the supersonic stream and its behavior both after reflection and after entry into the subsonic zone is calculated. The calculations are carried out in terms of the Mach numbers and adiabatic constants (ratios of specific heats) of the two streams.

The problem was formulated as a preliminary investigation of the interaction between shock waves originating in a supersonic stream and the subsonic flow in an adjacent boundary layer. The results of this paper appear to indicate that the shock wave may produce a retarding effect both upstream and downstream of the shock wave. M. J. Thompson, USA

111. K. P. Stanyukovich, Self-similar flows of a ponderable gas (in Russian), Doklady Akad. Nauk SSSR 64, 467–470 (Feb. 1949).

The equations for spherical waves in a compressible ponderable medium are written in the form

$$r^{-2}[r^2(u_t + uu_r + pr/\rho]_r + 4\pi G\rho = 0,$$

$$(\log\rho)_t + u(\log\rho)_r + u_r + 2u/r = 0,$$

$$(\log p)_t + u(\log p)_r + k(u_r + 2u/r) = 0,$$

where p is the pressure, ρ density, u radial velocity, G the gravitational constant, and k the ratio of specific heats. The following types of "automodel motions" (corresponding to what is called "progressing waves" in Courant and Friedrich's *Supersonic Flow and Shock Waves*, New York, 1948) are investigated:

(1) $u = x\,r/t$; $p = y\rho\,r^2/t^2$. Partial differential equations are obtained for x, y and ρ; these are transformed into ordinary differential equations by assuming that x, y and $\eta = \rho t^2$ are functions of the single variable $z = rt^{-a}$. The special cases $z = t$, $z = r$ are considered.

(2) $u = r/t$. It is found that p and ρ become of the form Ar^m/t^n, with certain relations between the exponents and the factors A.

(3) $u = 0$. It follows that p and ρ are functions of r only. This leads to an equation of Emden's type for gaseous spheres.

(4) $u = xr$; $p = y\rho r^2$. A solution is sought in which x, y and ρ are functions of the single variable $z = re^{-at}$.

J. M. Burgers, Holland

112. J. B. Zeldovich and K. P. Stanyukovich, On the reflection of a plane detonation wave (in English), C. R. Acad. Sci. URSS, 55, 587–590 (1947).

For a specific heat ratio of 3, the problem of one-dimensional reflection of a detonation wave from a rigid wall can be solved in closed form [see K. P. Stanyukovich, same source, 53, p. 523, 1946]. This solution is applied to the case of a stick of explosive abutting against a wall and ignited at the free end. The total impulse, the pressure-time and the velocity-time curves are given. Then the results for the case of instantaneous explosion (the ratio being still 3) are briefly stated. A. W. Wundheiler, USA

113. M. J. de Schwarz, Application of the method of the acceleration potential to the calculation of aerodynamic unsteady forces in supersonic regime (in French), Off. nat. Étud. Rech. aéro. Rep. no. 25, 31 pp. (1949).

The author applies the acceleration-potential method to the study of the aerodynamic forces on thin airfoils in unsteady supersonic stream in a way similar to the one which was used before by C. Possio (not mentioned in the bibliography). He considers first of all the case of harmonic oscillations, and after deriving the integrodifferential equation which defines the distribution law of the pressure doublet on the airfoil, he solves this equation with the help of a Laplace transformation. He extends afterwards the procedure to the general case and succeeds in such a way in solving the integrodifferential equation by means of a double Laplace transformation. Carlo Ferrari, Italy

114. F. Tricomi, On linear partial differential equations of the second order of mixed type, Hdqtrs. Air. Mat. Comm. Dayton Transl. no. A9-T-26, 186 pp. (1948).

This is a translation of the celebrated 1923 paper (R. C. Accad. Lincei, ser. 5, 14, p. 134) basic to transonic flow theory. Ed.

115. Y. H. Kuo, On the hodograph method, J. Aero. Sci. 16, 189 (1949).

This note contains the author's answer to the criticism by Lighthill [Proc. roy. Soc. London Ser. A. 191, 323–369 (1947); see Rev. **994,** June 1948] and Cherry, [Proc. roy. Soc. London Ser. A 192, 45–79 (1947); see Rev. **870,** May 1948] of his paper with Tsien [Nat. adv. Comm. Aero. tech. Note no. 1063 (1947)].

L. Bers, USA

116. J. W. Craggs, The breakdown of the hodograph transformation for irrotational compressible fluids flow in two dimensions, Proc. Camb. phil. Soc. 44, 360–379 (July 1948).

In the first half of the paper the author considers singularities of general mappings between arbitrary (x,y) and (u,v) planes such that u and v are analytic functions of x and y. A careful, almost exhaustive, mathematical classification of the various possible

subcases of singular points and curves of order one (at which the Jacobian $J = \partial(u,v)/\partial(x,y) = 0$ while J_x or $J_y \neq 0$) and of order two (at which $J = J_x = J_y = 0$ while at least one of the second partial derivatives of J does not vanish) is presented together with graphical illustration of the mapping of curves passing through the singularities. In the second half of the paper the mappings are specialized to those between the physical and hodograph planes (and their inverses) corresponding to possible irrotational compressible fluid flows in two dimensions. With the help of the general theorems established in the first part, the possible singularities of the flow mappings are described. Special attention is paid to the features distinguishing the limiting lines (ψ_q, ψ_θ remain finite; the line is an envelope of Mach lines) from the "branch lines" (ψ_q, $\psi_\theta \to \infty$; the line is a Mach line). It is shown that a branch line (also known as transition line) must occur in any transonic flow through an unbent nozzle, thereby causing a breakdown of any series representations. The results of the study overlap with those obtained independently in Courant–Friedrichs, *Supersonic flow and shock waves*, pp. 63–69 and 257–259.

M. V. Morkovin, USA

117. L. I. Sedov, **Hydroaerodynamic forces on a streamlined profile in a compressible fluid** (in Russian), Doklady Akad. Nauk SSSR 63, no. 6, 627–628 (Dec. 1948).

The barotropic flow past a cascade of obstacles is considered. The lift on a single member of the cascade is computed in terms of the circulation. The limit case is considered where the gap tends to infinity. [The conclusion thus reached, that the Joukovsky formula is valid for barotropic flows past a single airfoil, does not follow (e.g., note that the flow at ∞ is not regular in this limit case) and the question of the validity of the result remains open.]

G. F. Carrier, USA

118. **Bernard N. Daley and Lillian E. Hanna, Subsonic two-dimensional-flow conditions near an airfoil determined by static pressures measured at the tunnel wall,** Nat. adv. Comm. Aero. tech. Note no. 1873, 45 pp. (Apr. 1949).

The velocity field around two airfoils was determined by experiments in the Langley rectangular high-speed tunnel (Mach number 0.3 to 0.8). The measurements were made with the help of numerous orifices in the model end plate which was fitted flush with the tunnel wall. The knowledge of such field distributions is useful for the interpretation of schlieren photographs, the study of wind-tunnel constriction and interference problems.

The authors use these experiments to show that the following computing method gives reliable results for low Mach numbers: From the velocity distribution on the surface of an airfoil for $M = 0$ the velocity field around it is computed with the help of the Cauchy contour integral. The corresponding field distributions for low Mach numbers are computed with the help of the Prandtl-Glauert rule.

I. Flügge-Lotz, USA

119. **S. V. Falkovich, The lift force of a wing of finite span in supersonic flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1205-IA (GDAM A9-T-23), 11 p. (Jan. 1949): see Rev. 86, Jan. 1949.

120. **C. C. Lin, E. Reissner and H. S. Tsien, On two-dimensional non-steady motion of a slender body in a compressible fluid,** J. Math. Phys. 27, 220–231 (Oct. 1948).

In this paper the authors present a simple unified treatment of the unsteady two-dimensional polytropic potential flow about a slender body. It is shown that perturbation theory allows most problems to be reduced to five basic cases. Let δ be the larger of the following two quantities: the thickness ratio of the body and the ratio of the amplitude of lateral motion of the body to its chord; and let M_∞ designate the Mach number at infinity and $1/k$ be the characteristic period of time for transient motions.

The following cases are then distinguished: (1) The transonic case in which $M_\infty - 1 = O(\delta^{2/3})$, $k = O(\delta^{2/3})$, $\delta \ll 1$. (2) The subsonic or supersonic case in which $|1 - M_\infty{}^2| \gg \delta^{2/3}$, $k = O(1 - M_\infty{}^{-2})$, $\delta \ll 1$, $\delta M_\infty \ll 1$. (3) The highly oscillatory cases in which $k \gg \delta^{2/3}$, $k \ll |1 - M_\infty{}^{-2}|$, $\delta \ll 1$, $\delta M_\infty \ll 1$ and (4) $k^{-1} = O(M_\infty)$, $k\delta M_\infty \ll 1$. (5) The quasi-stationary case of nearly incompressible flow in which $kM_\infty \ll 1$, $k\delta \ll 1$.

The corresponding partial differential equations and boundary conditions are discussed, and appropriate similarity laws established. The equation describing case (1) is nonlinear, while the other four cases are linear. The authors have thus obtained specific answers to the question of the conditions under which the linearized perturbation theory may be applied when the value of M is in the neighborhood of unity.

Stefan Bergman, USA

121. **L. A. Galin, Remarks on the wing of finite aspect ratio in supersonic flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1218-IA (GDAM A9-T-37), 9 pp. (May 1949): see Rev. **532,** Mar. 1948.

122. **S. V. Falkovich, On the theory of a wing of finite aspect ratio in supersonic flow,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1215-IA (GDAM A9-T-34), 9 pp. (May 1949): see Rev. **881,** May 1948.

123. **I. E. Garrick and S. I. Rubinow, Flutter and oscillating air-force calculations for an airfoil in a two-dimensional supersonic flow,** Nat. adv. Comm. Aero. Rep. no. 846, 25 pp. (1946, issued in 1949).

The method of Possio which is based on solving the differential equation of the velocity potential for small disturbances is described and then applied to the problem of determining the aerodynamic forces in a two-dimensional supersonic flow which act on an airfoil oscillating with constant amplitude in simple harmonic motion. Theoretical expressions which involve integrals of Bessel functions are developed for the aerodynamic lift, pitching moment, and flap-hinge moment in the degrees of freedom of airfoil vertical translation, pitch, and flap rotation. A recursion formula is shown by which computations of these aerodynamic coefficients are appreciably simplified. Numerical tables are also presented by means of which the aerodynamic coefficients for the vertical translation and pitching degrees of freedom can be fairly easily computed for Mach numbers of 10/9, 5/4, 10/7, 5/3, 2, 5/2, 10/3, and 5 and for a fairly wide range of reduced frequencies.

These results are next applied to the flutter problem involving vertical translation and pitching of two-dimensional unswept airfoils at supersonic speeds. Graphs showing the dependence of the critical flutter speed on various parameters of the airfoil are included. The important results based on this idealized two-dimensional treatment at supersonic air speeds show that: (a) flutter is restricted to a more narrow range of airfoil parameters at supersonic airspeeds than at subsonic airspeeds; thus a given airfoil may flutter at subsonic speeds but not at supersonic speeds; (b) for normal cases when the airfoil bending frequency is low compared to the torsion frequency, flutter is not likely if the airfoil center of gravity is kept forward of the airfoil midchord; for the case when the bending frequency equals the torsion frequency, the center of gravity should be kept forward of the elastic axis in order to prevent flutter; the critical flutter speed tends to become a

minimum as the ratio between the bending frequency and torsion frequency approaches unity.

The report concludes with brief discussions of wing divergence, aileron reversal, and oscillatory instability in the one degree of freedom involving the airfoil pitch. It is pointed out that a similar instability does *not* exist in the one degree of freedom involving bending. Benjamin Smilg, USA

124. P. Blanc, Introduction to the study of jet propulsion, Part I, Mém. Art. Française: 20, 877–1004 (1946); 21, 885–1006 (1947).

This is an elementary and highly detailed discussion of the scalar properties of one-dimensional duct flow with heat transfer and chemical changes. The usual tables and graphs for numerical work with ducts are given, and their use illustrated by means of many examples. Ed.

125. J. E. Verschaffelt, Thermomechanics of ideal fluids (in French), Bull. Acad. Belg. Cl. Sci. 34, no. 4, 325–343 (1948).

Consideration is given to the dynamic action in an ideal fluid of a temperature gradient in one-dimensional flow. Assuming this gradient to be maintained in the absence of viscosity or thermal conductivity, it is shown that a flow is induced in the direction opposing the gradient. Extensions of the conventional dynamic and thermodynamic equations of fluid flow are developed to include this thermal force. Newman A. Hall, USA

See Rev. 1065 (Aug. 1949) for I. Prigogine's objections. Ed.

126. J. E. Verschaffelt, Thermomechanics of real fluids (in French), Bull. Acad. Belg. Cl. Sci. 34, no. 4, 344–355 (1948).

The action of thermal forces in real fluids is discussed, extending the corresponding ideal fluid theory (see preceding review). Restricting consideration to steady laminar flow, the action of an impressed thermal gradient is shown to be similar to that of impressed force fields. The effect of this thermal field on certain integrals of the equations of motion is indicated.

Newman A. Hall, USA

127. M. V. Blanc, P. G. Guest, Guenther von Elbe, and Bernard Lewis, Ignition of explosive gas mixtures by electric sparks, J. chem. Phys. 15, 798–808 (Nov. 1947).

In the first part the authors describe the apparatus and experimental procedure to measure the capacitances and gap voltages of condensed electric-spark circuits for sparks just powerful enough to ignite various explosive gas mixtures.

In the second part they develop an approximate theory of flame propagation from an instantaneous point source of ignition. The approximation essentially consists in considering the temperature gradient in the combustion front to be partly linear. The heat-balance equation, assuming spherical propagation, makes it possible to derive the distribution resulting from the temperature and the energy when one knows the diameter of the flame, the burning velocity, the initial temperature, as well as the heat conductivity, the density and the specific heat of the unburned gas.

Carlo Ferrari, Italy

128. J. Neustein and L. J. Schafer, Jr., Comparison of several methods of predicting the pressure loss at altitude across a baffled aircraft-engine cylinder, Nat. adv. Comm. Aero. Rep. no. 858, 15 pp. (1946).

In this paper four previously unrelated methods of predicting the pressure loss across a cylinder of an air-cooled reciprocating engine operating at altitude are correlated both analytically and experimentally. The analytical study indicates that the four methods, two of which are based upon empirical results and two upon theoretical considerations of one-dimensional compressible flow, are fundamentally related. The experimental investigation which consists of tests of a typical, baffled cylinder under various conditions of altitude, pressure drop, and heat transfer, shows that the relatively simple empirical method based on the density at the baffle exit gives results close to those of the two theoretical methods. It is found that accurate predictions of pressure loss at altitude can be made by any one of these three methods provided the pressure-drop coefficients used in the calculations are evaluated from sea-level tests *with heat transfer*. The second empirical method, based upon a characteristic Mach number, was found impractical because of the difficulty of defining a suitable characteristic state. Walter Vincenti, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 29, 110, 126)

129. Friedrich-Wilhelm Scholkemeier, Solution of Prandtl's boundary layer equations by means of power series (in German), Arch. Math. 1, no. 4, 270–277 (1948/49).

According to Frösling [Lunds Universitet Arsskrift N.F. Avd. 2 36, p. 32, 1940], the boundary-layer equations for flow about a rotationally symmetric body can be reduced to a partial differential equation for a single-stream function. Expanding in the usual way this function and the prescribed free-stream velocity in a power series of the variable x, measured along a meridional section of the wall, an infinite set of ordinary differential equations for the coefficients in the power series as functions of the coordinate perpendicular to the wall is obtained. The coefficients of the first five powers of x have been calculated by Frösling. The present author calculates the functions occurring in the coefficient of x^5 by a numerical integration of the corresponding differential equations, using Adam's procedure. R. Timman, Holland

130. J. Kampé de Fériet, On the flow of a viscous incompressible fluid between two infinite parallel plates (in French), Houille blanche 3, 509–517 (Nov.–Dec. 1948).

The author considers the turbulent flow between two parallel and fixed plates, $y = a$ and $y = -a$. If one assumes that (1) the trajectories of the mean motion are straight parallels to Ox, (2) the mean motion is steady, (3) the motion does not depend on z, and (4) Reynolds stresses do not depend on z and x, then the Reynolds equations reduce to two equations which contain $\bar{u}$, $\overline{u'v'}$, $\overline{v'v'}$ and $\bar{p}$. Each of these equations can be integrated once after introducing dimensionless quantities, one obtains the following two equations:

$$r - 1/R_* \cdot dV/d\eta = \eta, \tilde{\omega} + \sigma = -\xi,$$

where $\xi = x/a$, $\eta = y/a$, $R_* = u_* a/\nu$, $U_* = (\tau_0/\zeta)^{1/2}$, τ_0 is the intensity of the wall friction, $\bar{u}/U_* = V(\eta, R_*)$, $(\bar{p} - \bar{p}_0)/\rho U_*^2 = \tilde{\omega}(\zeta, \eta, R_*)$, $\bar{p}_0$ is the mean pressure for $x = 0$ on the fixed plates, $\overline{u'v'}/U_*^2 = r(\eta, R_*)$, $\overline{v'v'}/U_*^2 = \sigma(\eta, R_*)$. The author adjoins to the equations the boundary conditions and the assumption that the mean motion is symmetric about the plane $y = 0$, and finds an expression for the law of resistance, which depends only on the correlation coefficient r. It is impossible to obtain any other results from Reynolds' equations, and in order to proceed one must make new hypotheses. The author shows that the assumption that a universal velocity curve exists [$V = \phi(U_* \delta/\nu)$, δ is the distance to the wall] and the law of Stanton [$(\bar{u}_{\max} - U_0)/U_* = \text{const.}$; U_0 is the mean value of $\bar{u}$ over a section] suffice to express the reduced velocity curve V and the law of resistance in the term of Kármán's logarithmic functions. Finally the author shows that a

universal velocity curve cannot satisfy Reynolds' equations and boundary conditions. The assumption that an under-laminar layer exists is not sufficient to overcome this impossibility, since the existence of the universal velocity curve makes the correlation function r discontinuous in the $y = 0$ plane.

Ratip Berker, Turkey

131. R. Ramsauer, **Physical problems of boundary-layer friction, I: Connections between the friction coefficient and normal load in boundary-layer friction** (in German), Kolloid Z. 3, 83–95 (Nov. 1948).

This is the first of three papers on the subject of boundary-layer friction, all of which will be of interest. In the first section the author applies the work of Holm, Bowden and others on dry friction to the case of friction between solids with oriented surface layers. On the basis of the fact that the force of friction is proportional to the actual contact surface of the two bodies, the physical phenomena in boundary-layer lubrication are correlated with the physical phenomena in surface deformation. The experimental results confirm the theory that the friction at light loads is due to surface formed by elastic deformation, but at heavier loads plastic deformation determines the relevant surfaces.

Erle I. Shobert II, USA

132. H. Görtler, **Frictional resistance of a slightly corrugated plane parallel to direction of flow** (in German), Arch. Math. 1, no. 6, 450–453 (1948/49).

In two papers published previously [Z. ang. Math. Mech. 25/27, 223–244 (1947), and 28, 13–22 (1948)], the author investigated the flow pattern of a stationary, plane, laminar boundary layer along a plate of a periodic wave surface. In that discussion the undisturbed laminar boundary layer along a flat plate was assumed to be the basic flow pattern, and all the results were referred to the characteristic parameters of that boundary layer. The perturbation method was used and the equations were linearized with respect to a perturbation parameter ϵ. The quantity ϵ was assumed to be equal to a dimensionless measure of the surface wave length. But the deviation of the value of the friction drag of a wave-surface plate from that one of a flat plate is a perturbation effect of the second order in ϵ. In the present note the author calculates the friction drag of a wave-surface plate in reference to the friction drag of a flat plate. An interesting result is that the drag of a wave-surface plate is smaller than that one of a flat plate, which is in full agreement with numerical calculations published by Quick and Schröder. M. Z. Krzywobłocki, USA

133. E. Hardtwig, **The hydrodynamic equation of mean turbulent flow** (in German with Russian summary), Z. angew. Math. Mech. 28, 290–296 (1948).

The fundamental equations for the mean motion in a turbulent flow are derived on the basis of four assumptions. The final equations appear to be of the same form as those for laminar motions; the viscosity coefficient is replaced by a much larger exchange coefficient. C. C. Lin, USA

134. S. D. Nigam, **On turbulent liquid motion outside a circular boundary,** Phil. Mag. 39, 873–877 (Nov. 1948).

Ray [Phil. Mag. 28, 231 (1939)] treated the problem using a numerical solution of the Taylor-Goldstein equations. He found a retrograde motion near the circular boundary, and a reversal to direct motion at about one-third of the radius from it. The author constructs an analytic solution of the same equations by taking a different form for the "mixture length." This solution does not take into account the viscous layer at the wall, but is adjusted to make $\partial u/\partial y$ infinite there. It breaks down where the wake is formed. The flow is no longer retrograde at the wall where the velocity turns out to vanish. Ed.

135. **Theodore von Kármán, Progress in the statistical theory of turbulence,** J. Mar. Res. 7, no. 3, 252–264 (Nov. 1948).

The author points out the most important steps in the development of the theory of turbulence to which he made contributions of fundamental importance. The foundations of the theory are due to Reynolds, who introduced the idea of molecular disorder. Prandtl's ideas on momentum transfer and Taylor's suggestions concerning vorticity transfer, represent a great progress in the period 1915–1932. The formulation of this problem given by von Kármán in 1930 has the advantage of being more general and independent of the methods of the kinetic theory of gases. Kármán's theory led to the discovery of the logarithmic law of velocity distribution for the case of homologous turbulence. Then Taylor introduced the definition of isotropic turbulence and simplified the problem by consideration of a uniform and isotropic field of turbulent fluctuation. In 1938 von Kármán and Howarth, and Taylor worked out the analysis of the correlation functions and spectral functions, quantities which are accessible to measurements in a wind tunnel.

The author believes that the principal aim of the period in which we find ourselves at present is to find the laws for the shapes of either the correlation or the spectral function in the case of Reynolds numbers which are not very small. He clearly defines the problem and points out the relations between assumptions and results. The results were compared with measurements carried out at the California Institute of Technology and it was found that the agreement is good, considering the assumptions made in the theory. J. Beránek, Czechoslovakia

136. **Stanley Corrsin and Mahinder S. Uberoi, Further experiments on the flow and heat transfer in a heated turbulent air jet,** Nat. adv. Comm. Aero. tech. Note no. 1865, 61 pp. (Apr. 1946).

Measurements have been made of the mean total head and temperature fields in a round turbulent jet with various initial temperatures. The jet spreads more rapidly as its density becomes lower than that of the receiving medium, even when the difference is not sufficiently great to cause measurable deviations from the constant density, dimensionless-dynamic-pressure distribution. The work represents an extension of the previous work of Corrsin [Nat. adv. Comm. Aero. adv. conf. Rep., no. 3123]. Inconclusive deductions concerning the ratio between eddy diffusivities for heat and momentum transfer are given.

Myron Tribus, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 86, 88, 98, 99, 103, 105, 118, 119, 121, 122, 154 155, 160)

137. **John Boshar and Philip Davis, Consideration of dynamic loads on the vertical tail by the theory of flat yawing maneuvers,** Nat. adv. Comm. Aero. Rep. no. 838, 5 pp. (1946, issued in 1948).

For a fighter-type airplane ($W = 8100$ lb), incremental vertical-tail load and load factor perpendicular to flight path are computed as functions of time t in the case of an empirically determined rudder deflection $\delta_r(t)$. The computations which follow conventional lines and use wind-tunnel data for the force derivatives wherever possible, show good agreement with load measurements in actual flight. Similar computations are carried out for a

large flying boat (W = 145,000 lb) in the cases of an abrupt and a sinusoidal rudder deflection. Here only a moderate amount of deflection suffices to produce the ultimate design loads on fin and rudder. It is felt that specifications of yawing maneuverability are necessary to determine critical loads on the vertical-tail structure. G. Kuerti, USA

138. William S. Aiken, Jr., Flight determination of wing and tail loads on a fighter-type airplane by means of strain-gage measurements, Nat. adv. Comm. Aero. tech. Note no. 1729, 1–23 (Oct. 1948).

The results of the analysis of loads measured during symmetric pull-up and push-down maneuvers in a specially instrumented fighter-type airplane are reported. The shear and bending moments of the wing panels and the horizontal tail were measured by means of strain gages. The lateral positions of the centers of pressure of the wing panels were deduced from these data at various values of Mach number, and satisfactory agreement with the results of wind-tunnel tests of a model of the airplane was obtained. No outboard shift of the lateral position of the center of pressure was observed with increasing Mach number up to M = 0.80. The measured horizontal-tail loads agreed well with loads estimated on the basis of wind-tunnel tests of the model. The tail-off aerodynamic-center position computed from the flight data was also in good agreement with the wind-tunnel data at low values of airplane normal-force coefficient.

The load carried by the fuselage was calculated by subtracting the sum of the measured wing and tail loads from the total normal force (computed from the measured normal acceleration). The fuselage lift so computed agreed with lift calculated on the assumption that this lift was equal to that which would be carried by the portion of the wing contained within the fuselage. All flight maneuvers were made at lift coefficients below the buffet boundary of the airplane. W. O. Breuhaus, USA

139. Courtland D. Perkins and Robert E. Hage, Airplane performance, stability and control, New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1949, viii + 493 pp., 269 figs. Cloth, 6.2 × 9.5 in., $7.

This book is written in the light of recent developments in high-speed aerodynamics, and appears to be an excellent and modern text and reference book on the design of high-performance airplanes. The authors have succeeded in presenting design information in practical and directly usable form but with sufficient theoretical background material to elevate the design problem above pure empiricism. The first half of the book includes: (1) a fairly thorough treatment of fluid mechanics and basic aerodynamics in the subsonic, transonic and low-supersonic ranges directed toward engineering application and design; (2) a chapter on propulsion including propeller and reaction systems; and (3) a chapter on performance calculations up to and including the low-supersonic range. The second half of the book deals with stability and control in the subsonic range, the authors having stated their belief that a treatment of these problems in the supersonic range would be premature. John E. Goldberg, USA

140. D. J. Lyons, Present thoughts on the use of powered flying controls in aircraft, J. roy. aero. Soc. 53, 253–292 (Mar. 1949).

After reviewing the limitations of fixed aerodynamic balance and the possibilities of aerodynamic servocontrols, the author discusses in terms of limits, methods, and stability the use of powered controls from the standpoint of obtaining operational reliability by (1) reversion to manual control in emergency, and (2) duplication of all parts of the control system. Design considerations in terms of performance, structural stiffness, pilot's operating forces, use with automatic pilot, and friction limits are examined. John M. Kopper, USA

141. A. J. Marx and J. Buhrman, Assisted control, II (in Dutch), Nat. LuchtLab. Amsterdam Rap. no. 1421, 5 pp. (Sept. 3, 1947).

(See Rev. **1052**, Aug. 1949.) The report contains an investigation of the influence of a spring-tab-equipped elevator on static longitudinal stability. It appears that in most cases stability with the stick fixed is reduced because of the action of the spring in the control mechanism. The influence of the spring tab on stick-free stability is negligible. Ernst Eujen, Germany

142. A. J. Marx and J. Buhrman, Assisted control, III (in Dutch), Nat. LuchtLab. Amsterdam Rap. no. 1431, 20 pp., 8 figs. (Dec. 31, 1947).

The report concludes a series of three papers (see preceding review) dealing with the effect of assisted control devices on the characteristics of longitudinal stability and maneuverability of airplanes. In the present report the influence of power-operated flight controls is examined.

At first the basic requirements for the control booster are stated. As a rule, it is assumed that "feel" in the controls of the airplane must be maintained although the advantages of control without feel are also considered. A discussion of several types of hydraulic and electric servomechanisms shows the different ways the above requirements can be fulfilled.

The influence of power-boost control on the flying characteristics of the airplane is dealt with in the following part. A theoretical discussion of the stick-free condition is given in the appendix. It is shown that, if well designed, the servocontrol does not introduce any unstability.

The advantages and disadvantages of the different ways of reducing the control forces, i.e. aerodynamic balanced elevator, spring-tab control, and servocontrol are compared. With respect to weight economy servocontrol is superior to the other methods, particularly in large airplanes. The comparison between hydraulic- and electric-power-operated controls shows no striking superiority of one method over the other.

Ernst Eujen, Germany

143. Martin Goland, Stick-fixed, short-period stability based on the Wagner air forces, Reissner Anniv. Vol., J. W. Edwards, Ann Arbor, 111–124 (1949).

An airfoil in uniform horizontal flight is subjected to a damped harmonic disturbance ($\lambda = -\alpha + i\omega$) which affects both vertical velocity w and angle of pitch ϑ. The resulting lift and moment are determined in terms of λ, $w(t)$ and $\vartheta(t)$, on the basis of nonsteady airfoil theory according to the formulation given by von Kármán and Sears (1938), using the approximate lift-deficiency function due to R. T. Jones. A conveniently simplified system of equations of airplane motion with unknowns w and ϑ is then considered and the nonsteady wing and tail lifts are introduced. Substitution of exponentials for w and ϑ yields a determinantal equation for λ. Only the root with the largest ω has physical significance; it is determined for two representative aircraft by iteration (I).

The same calculations are also performed under complete omission of the nonsteady terms (II), and under omission of the wake terms only (III). While the frequencies ω obtained are practically the same in all three cases, the damping rates α computed according to cases (II) and (III) are respectively 30 and 50 per cent larger than those in case (I); stability is thus overestimated. Contrary to expectation, method (II), which entirely disregards apparent mass and rotational damping, gives more

realistic results than method (III) which is more nearly equivalent to standard practice. G. Kuerti, USA

144. Lewis R. Fisher, Approximate corrections for the effects of compressibility on the subsonic stability derivatives of swept wings, Nat. adv. Comm. Aero. tech. Note no. 1854, 35 pp. (Apr. 1949).

The Prandtl-Glauert compressibility correction for three-dimensional flows, with the free-stream Mach number reduced on the basis of simple-sweep theory, is applied to the equations previously presented [T. A. Toll and M. J. Queijo, "Approximate relations and charts for low-speed stability derivatives of swept wings," Nat. adv. Comm. Aero. tech. Note no. 1581, 1948] in an attempt to estimate the first-order effects of compressibility on the stability derivatives of swept wings.

Gerald E. Nitzberg, USA

145. Langley Stability Research Division (Charles J. Donlan, compiler), An interim report on the stability and control of tailless airplanes, Nat. adv. Comm. aero. Rep. no. 796, 16 pp. (1944, publ. 1948).

There is nothing new in this report now [see J. K. Northrop, J. roy. aero. Soc. 51, no. 438 (1947); also Rev. **150** (1948)].

Edwin A. Kidd, USA

146. Leonard Sternfield, Effect of automatic stabilization on the lateral oscillatory stability of a hypothetical airplane at supersonic speeds, Nat. adv. Comm. Aero. tech. Note no. 1818, 1–37 (Mar. 1949).

Along with the increases in flight speed currently being obtained, the trend in aircraft design has been to higher wing loadings and greater sweepback (of wings). Among other things, this trend has made it progressively more difficult to obtain satisfactory lateral stability and control characteristics. In the subject paper the possibility of improving the lateral stability characteristics of such machines by means of automatic control response is investigated.

Four types of automatic control response are considered: (1) rudder deflection (or equivalent) excited by a displacement in yaw; (2) aileron deflection (or equivalent) excited by a displacement in roll; (3) rudder deflection excited by a yawing velocity; and (4) aileron deflection excited by a rolling velocity. The analysis follows conventional lines, the equations of motion in roll, yaw and sideslip being set up and the stability tests applied to the coefficients of the characteristic equation. In considering the added effect of automatic control response, only yaw due to rudder deflection and roll due to aileron deflection are included. The addition of these terms to the equation of motion leads to a fifth-degree characteristic equation, the stability tests for which are discussed. The results indicate that although all the aforementioned devices improve lateral stability, automatic rudder deflection in response to a yawing velocity is the most effective means, and in fact, was the only means of producing fully satisfactory results with the representative machine considered in the paper. C. B. Smith, USA

147. Lawrence J. Gale and Ira P. Jones, Jr., Effects of antispin fillets and dorsal fins on the spin and recovery characteristics of airplanes as determined from free-spinning-tunnel tests, Nat. adv. Comm. Aero. tech. Note no. 1779, 1–21 (Dec. 1948).

A summary is given of the effects of antispin fillets and dorsal fins on spin-recovery characteristics which were observed during tests of 21 different models in the NACA free-spinning tunnels. It was found that antispin fillets (narrow extensions of the horizontal stabilizer along the fuselage) effectively increased the tail damping ratio of the model, and had a favorable effect on spin-recovery characteristics (the magnitude of which was a function of the mass distribution of the model). It was found that dorsal fins generally had little effect on spin and recovery characteristics. W. O. Breuhaus, USA

148. Raymond A. Young, Helicopter engineering, New York: The Ronald Press, 1949, 225 pp., 163 figs. Cloth, 6.3 × 9.3 in. $10.

Part I of this new book covers the fundamental theory of helicopter flight, mechanics of the rotor system, basic design, performance evaluation and stability theory. Part II covers rotor-blade design and rotor-blade stress analysis, as well as such miscellaneous factors as flying qualities, transmissions, vibration and economics. The appendixes include a dictionary of nomenclature peculiar to helicopter engineering.

The book approaches the problem of helicopter design in a logical and rational manner, discussing thoroughly the necessary aerodynamics of rotating wings and the geometry and mechanics of the semi-rigid rotor, as well as the many factors involved in both horizontal flight and vertical flight. Much of the information is also presented in the form of useful charts which are directly applicable to helicopter-design problems.

John E. Goldberg, USA

149. G. J. Sissingh, Automatic stabilisation of helicopters, J. Helicopter Ass. Gr. Brit. 2, 25 pp. (Oct.-Dec. 1948).

The author notes that the helicopter with a single sustaining rotor is dynamically unstable in the low-speed range, and undertakes a simple analysis of several devices for automatic control. The first part of the paper is restricted to control displacements which are proportional to and in phase with the attitude of the helicopter, the angular velocity of the helicopter, and a combination of these. The first type gives a species of static stability, while the second damps the pitching or rolling moment. Any helicopter can be stabilized by a proper combination of these.

The paper describes and analyzes the Bell and the Hiller devices, and concludes that they are a good beginning but are not entirely satisfactory because they have too large automatic control displacements in phase with the angular velocity, which spoils both the dynamic stability and the control sensitivity. Contrary to fixed-wing aircraft, where a special gyroscope is required, the helicopter can make use of its rotor which is itself a gyroscope. If satisfactory automatic stability has not yet been achieved, it is not for lack of means but because the layout has not been right, due to overcontrol. The author expects new devices to appear in the next few years which will solve the problem completely. Alexander Klemin, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 30, 31, 104, 121, 122, 123)

150. D. B. Steinman, Problems of aerodynamic and hydrodynamic stability, Univ. Iowa Stud. Engng. Bull. no. 31, 136–164 (1947).

This paper discusses the self-excited oscillations of various engineering structures varying from submarine periscopes to suspension bridges. The significance of the Strouhal number is brought out and some interesting experimental data are presented regarding the oscillations of circular cylinders, with and without vanes, when towed through a hydrodynamic medium. It is shown that

the maximum amplitudes occur when the frequency of the shed vortices is near the natural frequency of the cylinder. The oscillatory instability of suspension bridges is then discussed in some detail by admittedly approximate methods based on static considerations of the aerodynamic lift and moment acting on the body, rather than by a true dynamic analysis. However the author claims to have obtained satisfactory checks between his theory and experimental model tests. It is pointed out that the oscillations can be eliminated or reduced by increasing the rigidity of the structure, increasing the mechanical damping opposing oscillations of the structure, and shaping certain cross sections of the body (for example, open slots in the roadway adjacent to main girders). The author emphasizes the need for developing a more rigorous hydrodynamic theory to insure safe designs.

Benjamin Smilg, USA

151. W. P. Jones, Negative torsional aerodynamic damping at supersonic speeds, Rep. Memo. aero. Res. Counc. Lond. no. 2194, 4 pp. (Sept. 1946; issued in 1947).

This paper has been made obsolete by such work as H. Cheilek and H. Frissel, "Theoretical criteria for single degree of freedom flutter at supersonic speeds," Cornell aero. Lab. Rep. CAL-7A (May 8, 1947), and C. E. Watkins, "Effect of aspect ratio on undamped torsional oscillations of a thin rectangular wing in supersonic flow," Nat. adv. Comm. Aero. tech. Note no. 1895. See following review.

Benjamin Smilg, USA

152. Charles E. Watkins, Effect of aspect ratio on undamped torsional oscillations of a thin rectangular wing in supersonic flow, Nat. adv. Comm. Aero. tech. Note no. 1895, 36 pp. (June 1949).

This report determines the supersonic aerodynamic torsional moment on a rigid rectangular wing of aspect ratio A oscillating in pitch about a spanwise axis at a distance x_0 from the leading edge. Solutions in integral form to the usual linearized supersonic potential-flow equation are found in two regions: the purely supersonic region outside the Mach cones from the wing tips, and the region within these cones. In obtaining these solutions it is assumed that the Mach cones from the tips do not intersect on the wing, although the author states that the final results are valid provided only that the Mach cone from each tip does not intersect the side forming the opposite wing tip. To investigate first-order effects of the frequency of oscillation ω the integrals in the exact solution are expanded in powers of ω and only the first powers retained. This simplification leads to a concise expression, in terms of the Mach number M, A, and x_0, for the component of the aerodynamic moment about the x_0-axis in phase with the pitching velocity. The dynamic stability in pitch of the wing is assured when this component is positive.

The results indicate that a decrease in A reduces considerably the range of values of M and x_0 for which negative values of aerodynamic torsional moment theoretically exist. That is, decreasing the aspect ratio of the wing has a stabilizing effect on the pitching oscillations of a rigid rectangular wing in supersonic flow.

Paul A. Libby, USA

153. I. E. Garrick, Bending-torsion flutter calculations modified by subsonic compressibility corrections, Nat. adv. Comm. Aero. Rep. no. 836, 6 pp. (1946, issued in 1948).

A number of calculations for wing bending vs. torsion flutter are carried out to study the effects of compressibility at subsonic speeds. Two-dimensional aerodynamic coefficients are used; in the compressible case, the aerodynamic terms are derived from Possio's development which permits only small disturbances to the main flow. The results show that for normal wings operating at subsonic speeds without shocks, the effect of compressibility on the flutter speed is quite small; however for "heavy" wings, the flutter speed may be reduced by as much as 15%.

Benjamin Smilg, USA

154. M. D. Haskind and S. V. Falkovich, An oscillating wing of finite aspect ratio in supersonic flow, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1219-IA (GDAM A9-T-38), 13 pp. (May 1949): see Rev. **353**, Feb. 1948.

Propellers, Fans, Turbines, Pumps, etc.

155. A. W. Vogeley, Calculation of the effect of thrust-axis inclination on propeller disk loading and comparison with flight measurements, Nat. adv. Comm. Aero. tech. Note no. 1721, pp. 1–17 (Oct. 1948).

Inclining the thrust axis of an airplane propeller away from the direction of flight causes an oscillating thrust component in each blade, and this thrust has an amplitude which increases with the angle of inclination. The author gives a simple formula for the primary thrust increase, and then compares calculated and measured values of "thrust-gradient coefficient" for a four-bladed propeller on a Republic Thunderbolt. The agreement is excellent in view of the simplifying assumptions. A good discussion is given of complicating factors and of ways to reduce the stress fluctuation.

W. C. Johnson, Jr., USA

156. Elliott G. Reid, The influence of blade-width distribution on propeller characteristics, Nat. adv. Comm. Aero. tech. Note no. 1834, 14 pp. (Mar. 1949).

Combined force and wake survey tests were made on a series of three-blade model propellers in order to determine the influence of blade-width distribution on efficiency characteristics. The blades of the model propellers had the same pitch distribution and activity factors, but differed widely in plan forms.

The results show that blades tapered from broad roots to narrow tips had higher efficiencies than blades of relatively uniform width at advance ratios below maximum efficiency values, and at power coefficients above 0.1. At power coefficients less than 0.1, however, the tapered blades were somewhat less efficient at all advance ratios. Wake-survey analysis indicates associated redistribution of blade loading for different operating conditions.

Incidental results of these investigations show definite interdependence of blade elements, and indicate the undesirability of incorporating highly cambered profiles in propeller blades.

Frank L. Wattendorf, USA

157. A. G. Smith, Heat flow in the gas turbine, Proc. Instn. mech. Engrs. 159, no. 41, 245–254 (1948).

The results of calculations of the Nusselt number for turbine and compressor cascades, based on the methods of Squire and Eckert, are compared with experimental values and with estimates obtained from Reynolds' analogy.

Radial temperature distribution in blades that are cooled at the root and in hollow blades cooled internally are considered in some detail. It appears that the effectiveness of root cooling is inversely proportional to blade size and is limited to blades made of high-conductivity materials. Internal air cooling requiring small quantities of air can be very efficient if the internal flow passages are sufficiently subdivided.

The fundamental relations of heat transfer in tubes and channels are also presented in connection with a discussion on heat-

exchanger performance. It is shown that regenerative-type heat exchangers may be less bulky than the recuperative type.
Andrew Féjér, USA

158. Robert O. Bullock, Ward W. Wilcox and Jason J. Moses, Experimental and theoretical studies of surging in continuous-flow compressors, Nat. adv. Comm. Aero. tech. Note no. 1213, 41 pp. (March 1947).

Experiments have been conducted to determine the conditions that cause surging in compressors and to determine the effect of various installations and operating conditions on the character of the velocity and pressure variations occurring during surging. These investigations were made on three compressor units and the variation of static, total, or velocity pressure with time was recorded. In addition to the experimental studies, a simplified analysis was made to determine how instability of flow may occur in a compressor. An examination, based on this analysis, was made of several possible methods of inhibiting the occurrence of surging.

Surging was found to be a periodic variation of pressures and velocities that may occur when the slope of the compressor characteristic curve is positive. A transition region of finite extent and characterized by erratic pulsations of small magnitude separated surging from stable operation. The frequency of the pulsations was always the same throughout the unit; the wave form was frequently nonsinusoidal.

The actual magnitude of the slope at which surging begins is determined by the time required for the static pressure in the external pipes to adjust itself to a change in flow conditions in the compressor passages. The magnitude and the frequency of the pressure pulsations, as well as the point on the characteristic curve at which surging begins, depend on a complex relation of the capacity and the resistance of each component of the total system. Because the occurrence and manifestation of unstable operation is dependent on the dynamic properties of the complete air-flow system, attempts to utilize one of the several methods for inhibiting surge should be made only on the actual installation with which the compressor is to be used. Authors' Summary

159. W. F. Schurts, Practical hydraulic coupling design modifications, Proc. nat. Conf. indust. Hyd. 66–77 (Oct. 1947).

The author outlines modifications of the coupling circuit that can be made to change the performance characteristics of an hydraulic coupling. Core or torus rings may be added to the circuit to secure better control of the fluid circulation which results in some improvement in torque capacity. A baffle plate is effective in reducing drag torque under stall conditions although some sacrifice in torque capacity is also made. A third modification consists in the addition of a deaerating chamber. An eccentric runner, and also an opening incorporated at the inner part of the circuit are illustrated as other methods for displacing the oil from the working circuit under stall conditions.

The writer's company has developed a modified circuit incorporating the proper restrictions of the area of the fluid circuit, with improved performance. In large diameter circuits, an effective method of securing a satisfactory reduction on drag torque under stall conditions is also obtained by increasing the number of blades in the circuit. The larger sizes of couplings are of spot-welded sheet-steel construction. W. W. Hagerty, USA

Flow and Flight Test Techniques

(See also Revs. 85, 127, 128, 194)

160. W. Perl and H. E. Moses, Velocity distributions on symmetrical airfoils in closed tunnels by conformal mapping, Nat. adv. Comm. Aero. tech. Note no. 1642, 58 pp. (June 1948).

The paper discusses the interference due to the wind-tunnel walls on the flow about an airfoil. The ratio of airfoil to tunnel size is, however, extremely large and represents only a very special case. The introduction to the report does, in fact, so state. It is unlikely that the calculations of the paper will find much application particularly at high air speeds. The method may, however, be found of use for similar problems involving smaller relative model size. John Stack, USA

161. Shirleigh Silverman, The determination of flame temperatures of infra-red radiation, J. opt. Soc. Amer. 39, 275–277 (Apr. 1949).

Nonluminous flame temperatures may be determined by means of a brightness thermometer in the infrared region. For validity, the radiating gases must be in thermal equilibrium, Kirchoff's law must apply, and a comparison black-body standard must be available. It was concluded that the 4.4-micron CO_2 band could be used to determine flame temperatures to within 5%. Experimental procedures for minimizing errors are briefly discussed.
K. W. Miller, USA

Thermodynamics

(See also Revs. 102, 125, 126, 127, 128, 157)

162. S. R. De Groot and M. Geldermans, The Joule-Thomson effect in ethylene (in English with French summary), Physica Hague 13, 538–542 (Nov. 1947).

The authors have calculated the Joule-Thomson coefficient of C_2H_4 from their own specific heat and compressibility measurements by means of the formula $\mu = (1/c_p)(T^2P)\partial(PV/T)_p/\partial T$, where the evaluation of the derivative was either done by differencing based on equidistant experimental data, or by differentiation of an empirical equation of state. The authors claim for this procedure an accuracy at least as good as with direct observation of the isenthalpic T, P variation. The calculations cover the range from 25 C to 150 C and from 1 atm to 2500 atm, and include a portion of the inversion curve. The results are presented in tables and graphs. Eric F. Lype, USA

163. J. O. Halford, Contribution of the deviation from perfect gas behavior to the entropy and heat capacity. Water and benzene, J. chem. Phys. 17, 405–408 (Apr. 1949).

As the author previously observed for the case of ethyl alcohol, the modified Berthelot equation is not at all satisfactory for the evaluation of the contributions of gas imperfections to the thermodynamic properties of water vapor. Agreement of the equation with available experimental data is fair, however, in the case of benzene and ethane. Serge Gratch, USA

164. John D. Stanitz, An analysis of the factors that affect the exhaust process of a four-stroke-cycle reciprocating engine, Nat. adv. Comm. Aero. tech. Note no. 1242, 35 pp. (Apr. 1947).

The author develops differential equations for the gas pressure in the cylinder during the two periods of the exhaust process when the ratio to exhaust pressure is either above or below the critical pressure ratio. He integrates both equations by an approximate stepwise method, overlooking the fact that the first equation, which covers most of the exhaust process, can be integrated in closed form. He discusses the effect of variation of the design parameters involved and concludes that those with a significant effect on

the exhaust process are: piston speed and area, initial pressure ratio and closing angle, flow coefficient, and nominal area of the exhaust valve. Graphs illustrate the effect of these factors on the cylinder gas-pressure ratio. Eric F. Lype, USA

165. Georges Claude and André-Georges Claude, On thermic utilization of soils (in French), C. R. Acad. Sci. Paris 228, 544–545 (Feb. 14, 1949).

A calculation of the maximum efficiency obtainable from a machine utilizing the vertical temperature gradient in the earth (0.03 degree centigrade per meter) is presented.

H. C. Brinkman, Holland

166. A. I. Khinchin, Mathematical foundations of statistical mechanics, New York: Dover Publications, Inc., 1949, 179 pp. Cloth, 5.1 × 7.5 in., $2.95.

This translation (from the Russian, by George Gamow) of an extraordinarily lucid book takes up the following topics: geometry and kinematics of the phase space (31 pp.), ergodic problem (26 pp.), reduction to the theory of probability (14 pp.), application of the central limit theorem (31 pp.), ideal monatomic gas (14 pp.), the foundations of thermodynamics (19 pp.), dispersion and the distributions of sum functions (18 pp.). An appendix contains a proof of the central limit theorem of the theory of probability. Ed.

167. J. de Boer and J. van Kranendonk, The viscosity and heat conductivity of gases with central intermolecular forces (in English), Physica Hague 14, 442–452 (Sept. 1948).

The coefficients of viscosity and heat conductivity have been calculated for gases consisting of molecules which interact according to an intermolecular potential field of the form: $\varphi(r) = 4\epsilon\,[(\sigma/r)^{12} - (\sigma/r)^{6}]$. The results are given in reduced form to allow a comparison to be made with all substances for which the constants ϵ and σ of the intermolecular field are known. The results are in agreement with the available experimental data for Xe, A, Ne, H_2, D_2 and He.

The deviations which appear for the light gases at low temperatures pointing to the necessity of applying quantum theory to the transport phenomena are discussed from a theoretical point of view, and are compared with previous theoretical calculations of the transport phenomena based on quantum mechanics. Ed.

168. Balbhadra Prasad, A theory of viscosity and thermal conductance of liquids and highly compressed gases, Phil. Mag. 39, 884–889 (Nov. 1948).

A brief semiempirical discussion of transfer phenomena in liquids yields Andrade's simple equation for viscosity and the suggestion that similar results should apply for heat conduction and for transfer properties in highly compressed gases.

Arthur Kantrowitz, USA

Heat Transfer; Diffusion

(See also Revs. 136, 161, 167, 168)

169. B. Thunell, Calculation of transient temperature distribution in slabs and cylinders of wood (in Swedish), Svenska TräforskInst. Medd. no. 14, 1–16 (1947).

This paper provides the theoretical background for heating processes occurring in the gluing and impregnating of wood. The author integrates the equation of heat conduction for the title cases, and shows the resulting temperature field by graphs. Analytical integration is used for homogeneous materials and Schmidt's graphical method for other cases. The problems discussed include steady and transient states, constant as well as variable boundary conditions and properties of material, heat sources with variable temperature, and the end effect in the case of a short cylinder. Eric F. Lype, USA

170. G. A. Omohundro, O. P. Bergelin, and A. P. Colburn, Heat transfer and fluid friction during viscous flow across banks of tubes, Trans. Amer. Soc. mech. Engrs. 71, 27–34 (Jan. 1949).

This paper describes the equipment and initial results of a cooperative test program undertaken to study heat transfer and fluid friction in commercial heat exchangers and in idealized models. The initial data are for an idealized shell-and-tube exchanger, and they are substantially in agreement with the findings of earlier investigators. However, it is expected that the wide range of test parameters subsequently to be employed will reveal more accurate methods of correlating the performance of this type of exchanger. H. G. Elrod, Jr., USA

171. Alfred Schack, New values and formulas for the radiation furnace gases (in German), Arch. Eisenhüttenw. 19, 11–16 (1948).

The measurements of Hottel and Egbert [Trans. Amer. Soc. mech. Engrs. 63, 297–307 (1941)] of the rate of radiation from carbon dioxide are discussed from the point of view of Tingwald's data [Phys. Z. 35, 715–720 (1934); 39, 1–6 (1938)] on the infrared absorption spectra of CO_2 and the author's own procedure for predicting total radiation intensities from spectral data [Z. tech. Phys. 5, 267–278 (1924)]. Within the range of temperature t, partial pressure p, and gas-layer thickness s, defined by 300 C $< t <$ 2000 C and $0.001 < ps$/atm-meter < 0.4, the agreement between experimental data and predicted radiation rates is generally better than ±10 per cent when the broadening of the absorption bands and the weakening of the extinction coefficients with increasing temperature are allowed for. An approximate formula, valid within a more limited range of t and ps, shows the rate of radiation to be proportional to the 3.2 power of the absolute gas temperature and to the 0.4 power of ps.

For water vapor, suitable spectral absorption data are not available. However, an approximate formula, which includes a correction for deviations from Beer's law below 650 C, agrees with Hottel and Egbert's total radiation data for this gas generally within ±10 per cent for $400 < t <$ 1900 C and $0 < ps < 0.36$ atm-meters. In thin layers CO_2 radiates more intensely than H_2O at the same temperature; the reverse is true for thick layers.

R. L. Pigford, USA

172. A. G. Ogston, The Gouy diffusiometer; further calibration, Proc. roy. Soc. Lond. Ser. A, 196, 272–285 (Mar. 22, 1949).

When a diffusing boundary is placed in the path of a beam carrying an image of an illuminated slit, this image is changed into a system of fringes which contracts as diffusion proceeds; this is the principle of the Gouy interference method for diffusiometry.

This method was described in a previous paper [C. A. Coulson, J. T. Cox, A. G. Ogston and J. St. L. Philpot, Proc. roy. Soc. Lond. Ser. A, 192, 382–402 (Feb. 18, 1948)]; it has also been studied independently by L. G. Longsworth for the experimental process and by Gerson Kegeles and Louis J. Gosting for a very complete theory [J. Amer. Chem. Soc. no. 10, 2510–2523 (Oct. 1947)].

In the present paper, the author, making use of the theoretical results of Kegeles and Gosting, derives a method of analyzing diffusion data given by a mixture of two diffusing substances;

this method has been tested with heterogeneous samples of lactoglobulin and with mixtures of lactoglobulin and sucrose or potassium chloride. A. Craya, France

Acoustics

(See also Revs. 190, 191)

173. **Pierre Liénard, On sonic pressures perceived by an observer in motion relative to a point source moving uniformly at a supersonic speed in a perfect fluid** (in French), C. R. Acad. Sci. Paris 228, 1108–1110 (Mar. 28, 1949).

In a previous paper [*same source*, 1949, vol. 228, pp. 910–912] the author derived expressions for the sound-field pressure due to a point source moving along the x-axis with a uniform velocity v, greater than the velocity of the waves in the medium. In this article the author has applied these results to the case of an observer moving relative to the source.

The sound pressure measured by the observer is found to be composed of two terms having different amplitudes and phases. Inside the acoustic cone, which is the envelope of the family of wave surfaces $(x - vt)^2 + y^2 - c^2t^2 = 0$, the moving observer will record two frequencies for each single frequency emitted by the source. However, if the observer moves along the surfaces of certain hyperboloids within the cone and, in the limiting case, on the surface of the cone, only a single frequency will be recorded. If the projection of the motion of the observer along the x-axis is uniform, then the frequency observed remains constant. Motion in the meridian plane still gives two frequencies even if the projection of the observer's motion along the x-axis is uniform. On the other hand, uniform motion along the x-axis reduces the two frequencies to constant values and simple functions of the Mach numbers, which one can easily calculate by measuring the two frequencies and the original frequency of the source.

N. Chako, USA

174. **Von Werner Schaaffs, On the velocity of sound in liquids as function of their structure** (in German), Z. Naturforsch. 3a, 396–405 (July 1948).

From a study of homologous series of organic liquids, the author observes a steady increase in the velocity of sound as the molecule of a given series is increased in size. For the asymptotic condition of large molecules filling all the space, the maximum velocity is 1562 mps for the liquids studied. The experimental results agree quite well with calculations based on applying to liquids the Van der Waals gas equation. Vincent Salmon, USA

175. **Russell W. Raitt, Sound scatterers in the sea,** J. Mar. Res. 7, no. 3, 393–409 (Nov. 1948).

Oscillograph records were made of sound which, after radiation into the sea, was scattered back to the source, presumably by biological organisms having density and compressibility close to that of sea water. Pulses 3 milliseconds long of 24 kc/s sound were radiated vertically downward. The numerous echoes received at the source due to backward scattering usually cause a reverberant effect, but under favorable conditions individual echoes can be observed. A statistical study was made of the acoustic size of the scatterers as a function of depth. It was found that most of the scattering in the top 180 meters of the sea off Lower California was due to scatterers having acoustic cross sections between 0.1 and 10 cm². Most of the larger scatterers were concentrated at the lower depths. In the layer from 20 m to 90 m below the surface, there were about 1.5×10^{-5} scatterers per cubic meter having acoustic cross sections greater than 1 cm², but from 90 m to 180 m below the surface the number was about 7×10^{-5} per cubic meter of sea water.

Richard K. Cook, USA

Ballistics, Detonics (Explosions)

(See also Revs. 88, 112)

176. **Fr. Poboril and Fr. Primus, Piercing of armor plates** (in French), Mémor. Artill. fr. 22, no. 86, 971–980 (1948).

The author derives an expression for the perforating velocity of cylindrical and ogival projectiles when fired against inhomogeneous armor plate. The inhomogeneity of the armor plate due to its being face-hardened is taken into account by the use of a mean Brinell hardness through the cross section of the plate. The resistance to penetration is assumed to be due to the Brinell hardness and to plastic flow of the plate material in front of the nose of the projectile. The velocities as calculated by this method differ at most only 9% from observed values.

Henry J. Barten, USA

Soil Mechanics, Seepage

(See also Revs. 37, 56)

177. **Albert Caquot and Jean Kerisel, Tables for the calculation of passive pressure, active pressure and bearing capacity of foundations** (in English), Gauthier-Villars, Paris, 1948, 120 pp. Paper, 9.9 × 6.5 in., 5 figs.

Tables and charts are presented for computing the active and passive earth pressure and the bearing capacity of soils possessing internal friction and cohesion if the surface of the soil is plane. The influence of surcharge is taken into account.

The results are valid not only within the zones of active or passive Rankine equilibrium, but also outside these zones. Hence, the information facilitates the computation of the pressure against retaining walls of such dimensions that their backs do not lie within a zone of Rankine equilibrium.

The theoretical developments are summarized in a previous paper by Dr. Caquot (Équilibre des massifs pulvérulents à frottement interne—Equilibrium of granular materials with internal friction, Gauthier-Villars, Paris, 1934). The present work has for its object the provision of convenient numerical results for use in applications. N. M. Newmark, USA

178. **A. Hrennikoff, Analysis of pile foundations with batter piles,** Proc. Amer. Soc. civ. Engrs. 75, 185–208 (Feb. 1949).

In this article the author presents a method of analysis of pile loads in foundations involving nonparallel piles. The method is illustrated by application to a retaining-wall foundation. Special features of the method are that it makes allowance for the lateral resistance of individual piles, and that it considers the horizontal foundation displacement in addition to the vertical.

The author also considers approximations in the application of his method. In the approximation which is apparently best adapted to design purposes, the only physical property of the piles and of the soil which is involved is the ratio of the allowable axial and lateral pile loads. C. Martin Duke, USA

179. **R. Dantinne, On soil-vibration measurements** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 3, 187–198 (1948).

An electromechanical seismograph is described. Soil vibrations caused by passing vehicles, heavy machinery, pile drivers, etc.

have been recorded with this instrument. The influence of vibrations on human beings (physiological effects) and on engineering structures (strength of materials) is discussed. Tables subdividing these effects into "dangerous" and "not dangerous" are set up. Finally various types of intensity scales, referring to displacement, velocity, and acceleration amplitudes are compared. The significant observation of predominant frequencies of the soils around 25 cps coincides with results reported by other investigators. R. K. Bernhard, USA

180. Morris Muskat, The performance of bottom water-drive reservoirs, Trans. Amer. Inst. min. metall. Engrs. 170, 81–111 (1947).

A theory is developed for wells and reservoirs producing by bottom water drives, on the assumptions: density differences between oil and water neglected; theory of homogeneous potential motion applicable to the two liquids; the same values of permeability to viscosity ratios for the water in water-saturated zones and for the oil in oil-saturated zones; pressure above the bubble point everywhere; steady conditions. The behavior of producing systems with different parameters such as well spacing $\hbar$, oil-saturated zone thickness h, vertical and horizontal permeabilities k_z and k_h, and for different well penetrations is represented through the "displacement efficiency" $E = V/ha^2$ and the "efficiency function" $F = EA^2$, where V is the volume of the pay per well that is flushed out by the time the water first enters the producing well, and $A = (a/h)\ (k_z/k_h)^{1/2}$.

The analytical problem is to find a distribution of the potential $\Phi = (p - \gamma z)/\mu$ (z = distance downward, μ oil viscosity), which satisfies the equation $k_h(\partial^2\Phi/\partial x^2) + k_h(\partial^2\Phi/\partial y^2) + k_z(\partial^2\Phi/\partial z^2) = 0$, so that: $\partial\Phi/\partial z = 0$ for $z = 0$ (top boundary of the pay); $\Phi =$ const for $z = h$ (bottom boundary) and at the well surface, i.e. for $z \leqslant b$, $r = r_w$ (b well penetration, r_w well radius). E and F have been calculated and plotted against A and for different values of $\rho\ \omega = (r_w/2h)\ (k_z k_h)^{1/2}$. If k_h equals or is little greater than k_z, the clean-oil production per well is independent of the well spacing within practical ranges of the latter and proportional to the cube of the pay thickness and to the ratio k_h/k_z.

When the anisotropy in the effective permeability is great, the clean-oil production per well increases with increasing spacing, and total production from a given area before water entry decreases with the number of wells. The often observed persistence of clean-oil production for long periods of time indicates that in such cases the effective k_z must be much less than $0.01k_h$.

The clean-oil production decreases with increasing well penetration; after the value corresponding to the displacement efficiency has been recovered, the water production develops at an increasing rate. With increasing well density the incremental increase in clean-oil production decreases, and optimum well spacing is determined by economic considerations.

Giulio de Marchi, Italy

Geophysics, Meteorology, Oceanography

(See also Rev. 179)

181. N. F. Barber and F. Ursell, The generation and propagation of ocean waves and swell, I. Wave periods and velocities, Phil. Trans. roy. Soc. Lond. Ser. A, no. 824, 527–560 (Feb. 24, 1948).

Measurements of the pressure fluctuations at the ocean bottom off Pendun and Perranporth were analyzed to find the periods of the surface waves approaching the coast of England. These periods were entered on a "propagation diagram" plotting time vs. distance from shore. Assuming that the periods are propagated with the theoretical group velocity, the previous positions of a wave of given period T_1 lie along a line of slope given by $gT_1/4\pi$ and of intercept on the time axis equal to the time of arrival of the wave. By comparing the source regions indicated by the propagation diagrams for particular examples of long-period swell with meteorological data showing the distances from shore of actual strong winds which would generate the waves, it was concluded that the classical theory is an adequate description of the actual behavior of the waves, i.e., that propagation is linear, with the waves moving with the group velocity appropriate to their period, independently of the influence of waves of other periods. The discussion of the observations, which includes detailed analysis of three situations, is preceded by a review of the theory, including a discussion of possible deviations from classical theory. The appendixes present (1) a demonstration that the pressure at the sea bottom due to a single regular train of waves is nearly sinusoidal, (2) a discussion of the theory of frequency analysis of the pressure records, and (3) a study of the optimum length and spacing of pressure observations for wave analysis.

M. Neiburger, USA

182. J. Proudman, On the mixing of sea water by turbulence, Proc. roy. Soc. Lond. Ser. A, 195, 300–309 (Dec. 1948).

The basic ideas of Sir Geoffrey Taylor concerning diffusion by continuous movement are applied here to the variations, both in time and space of the distribution of salinity in the sea. Under the assumption that an interval τ_0 exists such that for $t > \tau_0$, there is no correlation between the turbulent salinity at the time zero and either the turbulent displacement or the turbulent velocity of the same particle at time t. Formulas are developed for the distribution of salinity in terms of the distribution at an earlier time, and for the time rate of transport of mass of salt. These formulas show, that, in general, the coefficients of eddy diffusion, as usually defined, do not depend only on the state of turbulence, but also on the distribution of salinity. In certain simple cases, however, the coefficients of eddy diffusion do depend on the turbulence only; as an example, conditions in the central region of the Irish Sea are discussed. Also two other examples are worked out in which the distribution of the mean salinity at zero time is especially simple.

Paul Neményi, USA

183. O. V. Golubeva, On the simplification of hydrodynamic equations in investigations of surface currents (in Russian), Doklady Akad. Nauk SSSR 61, 453–456 (July 21, 1948).

The investigation leads to the integration of Navier-Stokes equations which is simplified by six assumptions; in particular (because of the thinness of the layer), viscous effects are introduced in the form of volume forces. The simplified equations permit the computation of velocities and pressures when the boundaries are known and viscous forces are assumed. The equations are integrated assuming a zonal circulation of atmosphere over a sphere covered by water. In this case it is possible to obtain the kinematical picture of currents using a transformation indicated by the author; one of the streamlines of the intermediate solutions has to coincide with the contours of the continents. Georg P. Weinblum, USA

184. Donatien Cot, On a singularity of the English channel currents (in French), C. R. Acad. Sci. Paris 228, 1173–1176 (Apr. 4, 1949).

It is generally admitted that for a half-daily tide, the highest velocity of the current and the amplitude of the tide are both proportional to the coefficient C of the tide for the considered half day. It was however found in the neighborhood of Cherbourg that the amplitude was proportional to C, while the highest

velocity was proportional to $C^{1/2}$. The author shows that these results are due to the friction in the narrow and shallow waters of the British Channel. L. J. Tison, Belgium

Lubrication; Bearings; Wear

185. **R. L. Brooks and M. L. Atkin, Observation of the lubricating oil film between piston ring and cylinder of a running engine,** Counc. sci. indust. Res. Aero. Res. Rep. no. 42, 13 pp. (Sept. 1948).

This paper presents the second part of an investigation of the oil film between the top piston ring and cylinder wall of a single cylinder aircraft engine. The electrical method described in a first paper, in which the lubricant trace on the oscillograph screen was photographed, was improved.

In this part of the investigation the effects on the lubrication trace of various oils and additives were observed. A great number of traces are reproduced. They show, among others, the improvement in the lubrication trace by the addition of anti-wear type additives, with and without an oiliness agent. In general, physical, rather than chemical factors determine the breakdown of the film. D. DeMeulemeester, Belgium

186. **Arvid E. Roach, The load-carrying ability of hydrodynamic oil films,** Mech. Engng. 71, 293–296 (Apr. 1949).

The author presents a timely and comprehensive review of recent work on the load-carrying capacity of hydrodynamic oil films. The four possible criteria of load-carrying capacity, film rupture, film stability, film heating and film thickness are discussed inconclusively.

The classical view that improvement in surface finish leads to greater load-carrying capacity is contrasted with the considerable body of opinion which holds that roughened or otherwise imperfect surfaces may be capable of supporting greater loads than extremely smooth surfaces. It is concluded that the effect of surface condition requires thorough re-evaluation.

Recent work on the effect of variation in magnitude and direction of load is reviewed with clarity, and the various modes of oil films resonance are discussed. It is stated that there is as yet no comprehensive treatment of the interaction between oil film forces and transient elastic forces, and of their effect on rotating shafts. F. T. Barwell, England

187. **Robert L. Johnson, Max A. Seikert and Edmond E. Bisson, Friction at high sliding velocities of surfaces lubricated with sulfur as an additive,** Nat. adv. Comm. Aero. tech. Note no. 1720, 30 pp. (Oct. 1948).

This paper describes kinetic-friction experiments on steel surfaces lubricated with sulphur-containing lubricants. One surface is a flat rotating disk while the other is a sphere, elastically restrained to measure the friction. The spherical rider slides on a spiral plate so that contact always occurs with a virgin surface of the disk. The sliding speeds range from 50 to 8000 fpm and the loads from 269 to 1543 g (126,000 to 225,000 psi initial Hertzian pressure).

Under the specified experimental conditions it is found that, with cetane, $\mu = 0.4$ at slow sliding speeds but the friction rises above 1500 fpm, and reaches values of $\mu > 1$. This is larger than for unlubricated steel surfaces and is attributed primarily to the prevention, by the cetane, of the formation of surface oxides on the steel. In addition, it is considered that the cetane is oxidized or desorbed at these speeds. (Volatilization of the cetane is not considered a possible explanation.)

In the presence of sulphur lubricants, the friction and surface damage show a marked increase at speeds above about 1000 fpm. The extent of the lubricant failure depends somewhat on the load and the sulphur concentration of the lubricant, but the critical velocity at which failure first sets in is almost independent of these variables. Since the local surface temperatures depend mainly on the sliding speed and not on the load, this suggests that the effects are connected with chemical reactions at the high temperatures existing between the rubbing surfaces. The results support the view that the rate of chemical reaction is a limiting factor in the lubrication of rapidly moving surfaces by extreme pressure additives. D. Tabor, England

188. **Toshio Nishihara and Toshinori Kōri, Determination of the life of a roller bearing** (in Japanese), Trans. Soc. mech. Engrs. Japan no. 44, 13, 1–7 (May 1947).

The authors calculate the life of a roller bearing according to the theory of fatigue strength developed by Nishihara and Yamada [Trans. Soc. mech. Engrs. Japan, Feb. 1944, 10, no. 38, part 1, 23–27]. The relation between load and life is calculated numerically for two types of roller bearings, and compared with catalog data. S. Moriguti, Japan

189. **Dezso J. Ladanyi, Effects of temporal tangential bearing acceleration on performance characteristics of slider and journal bearings,** Nat. adv. Comm. Aero. tech. Note no. 1730, 32 pp. (Oct. 1948).

This paper provides a theoretical analysis of the effect of temporal tangential bearing acceleration on the general performance characteristics of slider and journal bearings. The theoretical assumptions and the various steps in the analysis are clearly stated. The investigation shows that acceleration of one of the moving surfaces produces a decrease in the load-carrying capacity of the bearing. This effect is mainly dependent on the ratio of acceleration to speed, and is more marked the higher the ratio. Consequently for a typical turbojet engine, where the acceleration is small compared with the speed, the effect is small. On the other hand, in the sliding of piston rings against the cylinder wall of a reciprocating engine the acceleration may be very large compared with the sliding speed, particularly near the top and bottom dead centers. Here the decrease in load-carrying capacity of the oil film between the rings and the wall may be so marked as to lead to breakdown of the lubricant film with increased metal-to-metal contact. Similar effects may be expected in the lubrication of gears. D. Tabor, England

Marine Engineering Problems

(See also Rev. 87)

Biomechanics

190. **Jozef Zwisłocki-Mościcki, Theory of the mechanics of the cochlea. Qualitative and quantitative analysis** (in German), thesis, Eidgenöss. Tech. Hoch., Zurich, 76 pp. (1948).

This thesis is concerned primarily with the dynamics of the human cochlea from the viewpoint of the interaction of the mass and damping of the ductus cochlearis with the hydrodynamics of the fluid, as characterized by the local impedance. A differential equation is deduced on the presumption of long waves and is shown to be "independent of the shape of the canal and the course of the vibrations with respect to time" (the reviewer is not yet convinced that such broad claims are strictly true). The velocity of propagation in the cochlea is found to rapidly decrease. No consideration of reflected waves is deemed necessary.

Existing theories (Helmholtz, Ewald, Ranke and others) are

compared by the author to experiment and found inadequate, while the author's theory is found to satisfy the data examined. The conclusions derived from his theory seem significant: "What is important is the occurrence of a flat local vibration maximum which with increasing frequency, becomes displaced from the apex of the cochlea towards its base and makes possible a *rough* resolution of the frequencies. It therefore appears necessary that there should be a finer frequency analysis in the *nervous apparatus*" (reviewer's italics). Once again, as in the optic nerve, and in McCulloch's pregnant suggestions on degradation of information, one finds degradation and simplification in the information channels, with the burden of analysis on the higher nervous system. It appears to be a pattern of frequent occurrence in the sensory apparatus.

It is concluded that the proper functioning of the middle ear is dependent on the high impedance of the entrance to the cochlea. This is indirectly substantiated by Esser's work. The paper constitutes a valuable contribution to the understanding of the ear.

Robert R. Williamson, USA

191. Martinus H. M. Esser, The mechanism of the middle ear, Bull. math. Biophys. 9: Mar., 29–40; June, 75–91 (1947).

In part I of this paper the middle eardrum, ossicles, and oval window are studied through an equivalent mechanical analog, the two piston and connecting lever system. The purpose is to study the "goodness" of this part of the ear, i.e., the efficiency as a conductor, as a function of frequency and the constants of the equivalent system.

The problem bears a direct relation to the servomechanism problem of defining the quality of the servo by its transfer function (Esser's efficiency) and would have profited by the use of servoanalysis techniques and language. An important distinction is that while the servo designer is concerned with obtaining the best servo for a given job and may change the system (analysis and synthesis), the biophysicist is concerned with what is (analysis only).

It is concluded that two inequalities may be used to define goodness of the middle ear, both normally satisfied by the normal ear, and that the theoretical effect of removal of drum and ossicles is in agreement with experiment. The velocity of propagation in the fluid is considered constant.

In part II the author establishes the validity of the foregoing treatment of the drum as a piston connected to a lever. Two approaches, starting with Helmholtz's hypotheses, one based on stretchable and the other on unstretchable radial fibers, lead to two equations describing the dynamics of the eardrum. The equilibrium configuration of the drum under a constant force on its center and pressure over its area is determined. The dynamical characteristics are then derived by considering small deformations due to changes in central force and pressures. There is an excellent statement of the anatomy of the drum, ideal for purposes of mathematical analysis. Robert R. Williamson, USA

192. N. Rashevsky, A note on the mathematical theory of vibrations of walls of blood vessels, Bull. math. Biophys. 9, 123–126 (Sept. 1947).

This paper uses the method of Rayleigh's theory of the vibration of jets to treat the vibrations of blood vessels. By a more exact mathematical treatment it checks exactly an approximate treatment by the same author; this is one of a number of similar papers by the author's school. It would seem to this reviewer that the approximation method has sufficiently demonstrated its truth and worth, and that if further effort is to be extended toward demonstrating its validity it should be directed to a general examination, rather than to specific cases. Much information for this general demonstration is now available in the literature of Southwell's relaxation methods, to which the author's approximation method is related as a special case.

Specifically this paper treats exactly an idealized blood vessel with axial symmetry and blood as an ideal fluid, within limits of small vibrations and no axial stretching, to the point of obtaining by Lagrange's method the differential equation governing the oscillations of the blood vessel; it then approximates to linear terms. Any representation of tissue external to the vessel is ignored.

This is a difficult and important problem, and it is proper to be concerned with the validity of approximations since so many must be made. Nevertheless, it is doubtful that obtaining approximately a differential equation and then solving exactly will yield results very different from obtaining an exact equation and making equivalent approximations in its solutions.

Robert R. Williamson, USA

193. H. D. Landahl, A note on the units of membrane permeability to water, Bull. math. Biophys. 10, 187–190 (Sept. 1948).

The author discusses the numerical equality of dimensionally different measures of permeability to water, the quality resulting from the unit density of water. He then defines h_w by $Q_w = h_w A(C_{we} - C_{wi})$, where Q_w is the rate of flow of water under the concentration difference $C_{we} - C_{wi}$, across an area A. By introducing the bulk modulus of water, he relates the permeability h_w to the "penetrability" ξ^* measured in $\mu^3/\mu^2/\text{min/atm}$ by $h_w = 0.034\ \xi^*$ cm/sec. Alston S. Householder, USA

194. John L. Stephenson, Theory of the measurement of blood flow by the dilution of an indicator, Bull. math. Biophys. 10, 117–121 (Sept. 1948).

This paper presents a theory for the interpretation of measurements of cardiac output by means of an indicator. Considering the blood flow from an artery through a capillary bed into a vein, let $f(t)dt$ represent the probability that an indicator particle leaving the artery at $t = 0$ appears in the vein during the time interval t, $t + dt$. Then if the concentration in the artery is $C_1(t)$, that in the vein is $C_2(t) = {}_0\int^\infty C_1(t - \eta)f(\eta)d\eta$. If the injection occurs at a constant rate m, just beyond that point at which C_1 is measured, then $C_2(t) = m\theta(t)/F + {}_0\int^t C(t - \eta)f(\eta)d\eta$, where F is the rate of flow, and θ allows for transients, increasing monotonically to 1. The transients will be damped and C_2 will approach $C_2(t) = m/F + C_1(t) - mb/V$. For nonconstant rates, f may be found by solving the integral equation $C_2'(t) = C_1(0)f(t) + {}_0\int^t C_1'(t - \eta)f(\eta)d\eta$. Alston S. Householder, USA